PROBLEMS OF THE

55th—27th INTERNATIONAL—RUDOLF ORTVAY

PROBLEM SOLVING CONTEST IN PHYSICS

31 January—10 February 2025

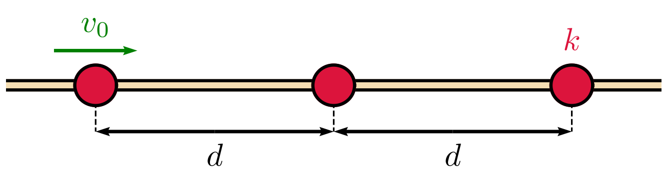
1. Imagine a world where the law of light reflection does not correspond to a perfectly elastic collision with a wall but rather to an *inelastic* collision with a wall, characterized by a coefficient of elasticity k. In this world:

   a) Write down the law of light reflection.
   b) Analyze the image formation by a plane mirror.
   c) Design a periscope.

   Point out as many differences and similarities as possible between the mirror optics of the real and the imagined world.

   (Zsolt Bihary)

2. On a horizontal, frictionless rod three small beads of identical mass rest at equal distances d apart, as depicted in the figure. At a certain moment, the bead on the left is given an initial velocity of magnitude v_0. Consequently, a series of inelastic collisions occur between the bodies, each of them characterized by the same coefficient of restitution $0 < k < 1$.



   a) How many collisions happen in total before the stationary state is reached? Plot the number of collisions as a function of the parameter k.

   b) Examine the limiting cases of perfectly elastic, $k \to 1$, and perfectly inelastic, $k \to 0$, collisions. Do we then recover the result of elementary considerations, that the total number of collisions is two?

   (Róbert Németh)

3. Let's make the realistic, but not permissible, assumption that we "cut the curve" with our car, that is, in order to increase the speed, we move into the oncoming lane for a short time while taking the curve.

   Furthermore let's assume that the radius of the road curve is r, after the bend our direction of travel changes by α, and the lane width is s.

   Relatively how much time can we gain with this, compared to staying in the lane, while in both cases we keep the lateral load on the tires at the same level below skidding? How does this ratio change if the angle of inclination of the track in the curve is β?

   How many milliseconds do we gain if $r = 50\,\text{m}$, $s = 3\,\text{m}$, $\alpha = 60^\circ$ és $\beta = 5^\circ$ and our maximum possible speed in the event that we had stayed in the lane would have been $v = 60\,\text{km/h}$?

   Is it worth the risk?

   (Oszvald Glöckler)

4. In the 18th century, Georges-Louis Le Sage proposed that gravity could be explained through collisions with hypothetical particles. Since two finite size material bodies partially shield each other from these colliding particles, an effective attractive force emerges between them. In this problem, we investigate the motion of the Sun—Earth system within the framework of this model.

   Let us describe the hypothetical particles using an ideal gas model, where the mean free path is much larger than the Sun—Earth distance. The collisions of the particles with the spherical Sun and Earth are characterized by the coefficient of restitution k. Assume a heliocentric framework: the Sun is "fixed"; and the average velocity of the particles is zero in the Sun's reference frame.

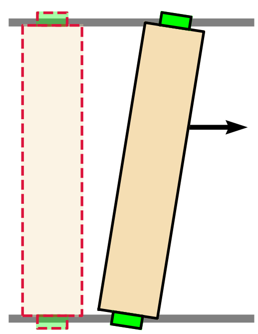
   a) What is the gravitational force acting on a stationary Earth located at a distance r from the Sun?

   b) How would the Earth move in this model if it were placed at a distance r with an initial tangential velocity v?

   (Gábor Széchenyi)

5. Consider a table drawer, of which one of its rollers has broken. Assume that the drawer is not sticking and that the friction on the unbroken roller is negligible. The two rollers have a common axis. The weight of the drawer is negligible.

   Consider the drawer, the rollers, and their guide rail as rigid bodies. It can be assumed that there is a very small gap between the rollers and the sides of their respective guide rails. The rail, as well as the force pulling the drawer, is x-directional. The line of force does not necessarily go through the center of mass. Describe the friction through the Coulomb model.



   Describe the movement of the center of mass, both when it is initially at rest and when it is not (which corresponds to the case of the roller breaking while the drawer was being opened).

   (Ákos Gombkötő)

6. A solid, cylindrical object has a density that varies linearly along the direction parallel to the axis of the cylinder. At one end of the cylinder, the density is ρ_0, while at the other end, it is ρ_1. The radius of the cylinder's base is $R = 2\,\mathrm{cm}$, and its length is $L = 20\,\mathrm{cm}$. The solid object is placed on the surface of water which has a density of $\rho_\mathrm{v} = 1000\,\mathrm{kg/m^3}$. Surface tension effects can be neglected.

   a) Using appropriately chosen metrics, determine the stable equilibrium position of the object if $\rho_0 = 100\,\mathrm{kg/m^3}$ and $\rho_1 = 1300\,\mathrm{kg/m^3}$. Calculate the frequencies of small oscillations around the stable equilibrium position. Drag forces can be neglected.

   b) How should the values of ρ_0 and ρ_1 be chosen if we want the average density to be $\rho_\mathrm{v}/2$ and the angle between the cylinder's axis and the water surface to be $2°$ in the stable equilibrium position?

   (Ádám Bácsi)

7. A pointlike mass is moving in three-dimensional space, characterized by position vector $\mathbf{r}$ and momentum vector $\mathbf{p}$. Its Hamiltonian is the following:

$$H(\mathbf{r},\mathbf{p}) \;=\; r\,\Phi(\mathbf{p}),$$

where $r = |\mathbf{r}|$ is the length (absolute value) of the location vector and $\Phi(\mathbf{p})$ is some differentiable function of the momentum vector.

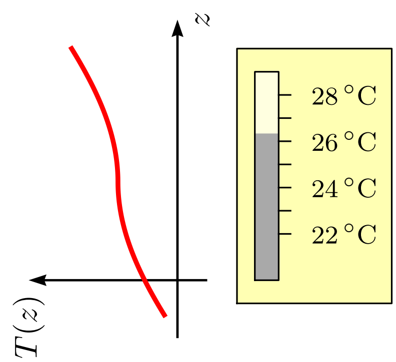
a) Derive the canonical Hamilton's equations for the object. Look for conserved quantities. Using these, derive the second-order differential equation of motion in the momentum space, which no longer contains the position coordinates.

Hint: it is advisable to introduce the notations $\mathbf{F} = \dot{\mathbf{p}}$ and $F = |\mathbf{F}|$.

b) Now let $\Phi(\mathbf{p}) = k\,p_z$, where k is a given constant. Derive and solve analytically the equations of motion of the object: determine all the components of the position vector $\mathbf{r}$ and the momentum vector $\mathbf{p}$ as a function of time. (*Hint:* in the first step, it is advisable to calculate the function $w(t) = 1/p_z(t)$). Give an outline of the movement trajectories that match the different initial conditions.

(Gyula Dávid)

8. There is a room where the temperature is not homogeneous, but depends on the height. It can be described by a known $z \mapsto T(z)$ function. In this room there is a mercury-in-glass thermometer, as it can be seen in the figure.



a) Which equation determines what the thermometer shows in equilibrium?

b) Assume that $T(z) = T_0 + Az$, with some constant T_0 and A. What temperature does the thermometer show?
We can neglect the heat transfer in the mercury column. The temperature of the mercury at z is $T(z)$. The coefficient of linear thermal expansion is α.

(Máté Veszeli)

9. Sound waves propagate isotropically in a medium, independently of frequency at a speed of $u < c$ (where c is the speed of light in vacuum).

a) Determine the dispersion relation $\omega(\mathbf{k})$ of the wave experienced by an observer moving at speed $\mathbf{V}$ relative to the medium.

b) Solve the problem in the case when the wave in the medium satisfies the Klein–Gordon equation with the limiting velocity u and the limiting frequency Ω.

(Gyula Dávid)

10. The half-space $z < 0$ is filled with a perfectly rigid material, in which no elastic waves can propagate. In the region $0 < z < h$ there is a homogeneous isotropic elastic material with density ρ_1, Lamé constants λ_1 and μ_1. The region $z > h$ is filled with another homogeneous isotropic material with density ρ_2 and elasticity parameters λ_2 and μ_2. The contact surface of the materials were glued together along the $z = 0$ and $z = h$ planes.

    a) A longitudinal elastic plane wave with frequency ω and amplitude A_0 arrives from $z \to +\infty$ at an angle ϑ with the z-axis.

    Calculate and plot the amplitude of the reflected wave(s) as a function of the angle of incidence.

    b) Calculate and plot the effective one-dimensional dispersion relation for such waves which are concentrated on the layer and propagating along the layer boundaries.

    Let us neglect the attenuation of the waves.

    (Gyula Dávid)

11. An infinitely long cylindrical cavity is carved into the homogeneous and isotropic elastic medium that fills the entire space. The tube is filled with another homogeneous and isotropic elastic material. The two materials are glued together on the contact surface. In the material inside of the cylinder, the propagation velocity of the longitudinal elastic waves is lower than that outside, but the speed of the transverse waves is the same.

    A longitudinal elastic plane wave arrives from infinity, its wave vector is perpendicular to the axis of the cylinder. The wavelength in both materials is much smaller than the diameter of the cylinder.

    Upon entering the cylinder, the wave is refracted, then passing through the core it reaches the surface again from the inside. There it is partially reflected, and partially refracted, eventually it exits the cylinder.

    What caustics are created by the refracted and one time reflected waves inside the cylinder? Give the equation of the resulting caustics and plot the caustics depending on the parameters of the problem!

    Note: if we continue to follow the path of the waves inside the cylinder, upon reaching the interface, another series of reflections or refractions follows. Let's not deal with these and other similar phenomena now!

    (Gyula Dávid and József Cserti)

12. Consider an infinitely long, one-dimensional crystal whose identical atoms of mass m are experiencing a Morse-type interaction. The Lagrangian takes the form

    $$L\left(\{x, \dot{x}\}\right) = \sum_{n=-\infty}^{\infty} \frac{m\dot{x}_n^2}{2} - \sum_{n=-\infty}^{\infty} \sum_{m=n+1}^{\infty} D\left[1 - \mathrm{e}^{-\kappa(|x_n - x_m| - a)}\right]^2,$$

    where D and κ are positive parameters, while a is the lattice constant describing the equilibrium state of the crystal. Determine the dispersion relation of longitudinal lattice vibrations and the speed of sound in this system.

    (Dmitry Zverevich)

13. A spherical ball with refractive index n is illuminated by a parallel, linearly polarized light beam. The radius of the sphere is many orders of magnitude larger than the wavelength of the light.

    a) What fraction of the energy of the incident light is reflected? Determine the reflected fraction as a function of the refractive index.

    b) Calculate and plot the angular distribution of the intensity of the reflected light in the case of two significantly different refractive indices.

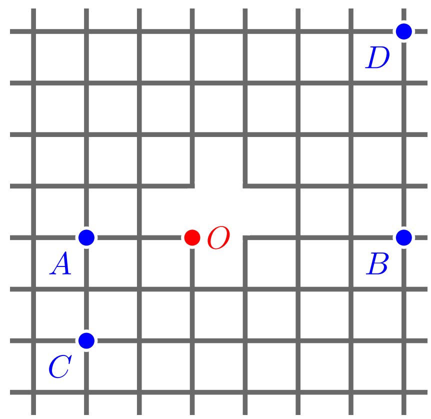
    (József Cserti)

14. Consider two ungrounded metal spheres, each with radius R and charge Q. What is the repulsive force between them if their centers are separated by a distance of $5R/2$?

(Gábor Széchenyi)

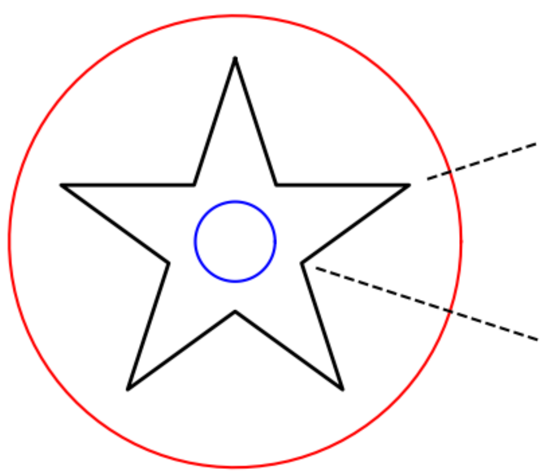
15. All edges of an infinite square grid have the same resistance. As shown in the figure below, four resistors are removed from this perfect resistor grid.

Find the analytic form of the resistance in this perturbed network between the lattice point O and each lattice point along a) the line segment AB and b) the line segment CD.



(József Cserti)

16. A constant direct current flows in the regular, five-pointed, star shaped frame shown in the figure.



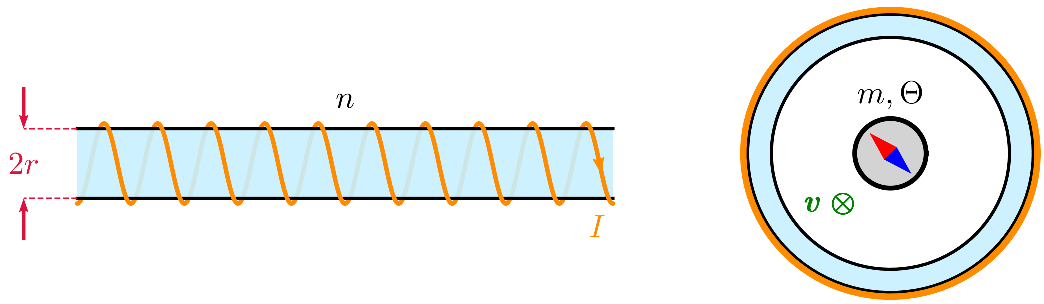
A magnetic dipole is located in the plane of the current frame, the magnetic moment of which is perpendicular to the plane. Calculate the a) force and b) torque acting on the dipole depending on the location of the dipole!

Plot the force vector acting on the dipole in the points forming a circle concentric with the star, if the circle is located c) outside the star (red circle), d) inside the star (blue circle).

e) Calculate and plot the magnitude of the force acting on the dipole along the radial line passing through the convex and concave peaks of the star (dashed lines), starting from a small distance from the frame to infinity. Determine the asymptotic form of the resulting function.

(József Cserti)

17. Consider an experiment carried out with a very long solenoid of radius r and turn density n in a region isolated from external magnetic effects. While keeping its current I at a stationary value, a small compass of dipole moment m and moment of inertia Θ is placed perpendicularly on the axis of the coil according to the figure. Initially at rest, the body is brought into motion at a certain moment with a uniform velocity parallel to the axis.



We experience that when this velocity is either very small or close to a critical value v_{crit}, the magnetic needle is rotating almost uniformly, otherwise a complex, hardly characterizable motion is observed. Explain the phenomenon. Express the velocity v_{crit} with the parameters given above and discuss its behavior as a function of the turn density n.

(Róbert Németh)

18. The Hamiltonian $\hat{H}$ of a quantum system of spin S is nonlinear but analytic function of the component $\hat{S}_z$ of the spin operator $\hat{\mathbf{S}}$.

a) Using Heisenberg picture give the components $\hat{S}_x(t)$ and $\hat{S}_y(t)$ of spin operator $\hat{\mathbf{S}}$ as functions of time t.

For the cases $S = 1/2$, $S = 1$ és $S = 2$ give the nonvanishing matrix elements of the operators $\hat{S}_x(t)$ and $\hat{S}_y(t)$ in explicit form.

b) The Hamiltonian of a more complicated quantum system depends not only on the $\hat{S}_z$ component of the spin operator $\hat{\mathbf{S}}$ but on the operator $\hat{A}$ of an additional physical quantity: $\hat{H} = \hat{H}(\hat{S}_z, \hat{A})$. The operator $\hat{A}$ has finite number of non-degenerate eigenvalues.

The commutators of the operators appearing are the following:

$$[\hat{H}, \hat{S}_z] = 0, \quad [\hat{H}, \hat{A}] \neq 0, \quad [\hat{S}_z, \hat{A}] = 0.$$

Using Heisenberg picture give the components $\hat{S}_x(t)$ and $\hat{S}_y(t)$ of spin operator $\hat{\mathbf{S}}$ as functions of time t.

Hint: Work in the subspaces belonging to the eigenvalues m $(-S \leq m \leq S)$ of the operator $\hat{S}_z$ and use the spin ladder operators $\hat{S}_+$ and $\hat{S}_-$. At first step it is worth calculating the commutators of the spin ladder operators $\hat{S}_\pm$ and an arbitrary analytic function $f(\hat{S}_z)$ of the spin component $\hat{S}_z$. Use the convention $\hbar = 1$.

(Gyula Dávid)

19. In the attached table and on the plot one can find data measured by a radon gas detector as a function of time. The device was used in a room with a high radon concentration in air ($510\,\mathrm{Bq/m^3}$) for a few days, then quickly moved to open air (outdoors), where the radon concentration is very small—this is when we started the presented measurement. The measured concentration values did not drop instantly.

Let us assume that the detector is not sensitive to radon, but only to its daughters. Let us also assume that the detector is either only sensitive to alpha decays, or it is only sensitive to beta decays. Questions:

a) Is the detector sensitive to alpha, or to beta decays?

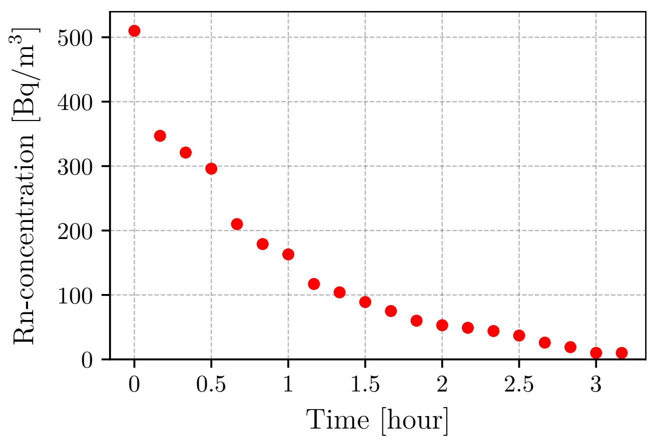
b) What is the relative efficiency of detection for the two relevant daughter isotopes?

c) What is the sensitivity of the device, i.e. how many counts does it register per hour, in case of a reference radon concentration of $37\,\mathrm{Bq/m^3}$?

d) The device is kept in open air for a few days, then moved back to the same room with the same high radon concentration. Let us make a plot of the time dependence of the value shown by the device.

The measured data, averaged over intervals of 10 minutes ($\mathrm{Bq/m^3}$):

510, 347, 321, 296, 210, 179, 163, 117, 104, 89, 75, 60, 53, 49, 44, 37, 26, 19, 10, 10.



(Gábor Veres)

20. An isolated ecosystem, containing exclusively generalist species is composed only of a productive (plant), a primary consumer (herbivore), and a secondary consumer (carnivore) level. Omnivores are not present.

To the great surprise of ecologists, both the productive and primary/secondary consumer levels are of constant value since time immemorial (although the ratio of the populations of species within a given level may change), except for some short-lived perturbations.

However, someone accidentally sprays a small amount of radioactive material over the area every day, causing a slight increase of the death rate.

We can assume that the population dynamics is continuous and deterministic, and it is modified by the surplus radiation in the following way:

$$\dot{N}_i = f(N_1, N_2, N_3) \quad \rightarrow \quad \dot{N}_i = f(N_1, N_2, N_3) - qN_i,$$

where q has the same value for all ecological levels. (N_i means the number of individuals in the i-th level.)

How does the stable equilibrium population change due to the radioactivity? At what value of the effective coefficient q does the stability of the state of the ecosystem cease?

(Ákos Gombkötő)

21. A charged particle of mass m is being trapped and shuttled with a quasi one-dimensional time-dependent electric potential. The shape of the well is unchanged and harmonic throughout the process, however, its minimum position is tuned according to a continuous function $t \mapsto x_0(t)$, that is,

$$V(x,t) = \frac{m\omega^2}{2}\left[x - x_0(t)\right]^2,$$

where ω is a characteristic angular frequency. The experiment is performed such that in both initially and after a long time, the well is in rest or moves uniformly, that is, $\ddot{x}_0(t \leq 0) = \ddot{x}_0(t \to \infty) = 0$. Furthermore, we know that at the beginning of the process, the particle was prepared in the ground state.

a) What is the probability of finding the particle in the current ground state at the end of the process, after a long time?

b) Examine the probability computed in *exercise a)* for a special case, a uniformly accelerated motion:

$$\ddot{x}_0(t) = \begin{cases} 0 & \text{if } \ t \leq 0, \\ a_0 & \text{if } \ 0 < t \leq T, \\ 0 & \text{if } \ t > T. \end{cases}$$

Interpret the results. Discuss the limiting case of $a_0 \to \infty$ and $T \to 0$ when $a_0 T = v_0$ is kept at a constant value.

c) Examine the probability computed in *exercise (a)* for another special case, a harmonic oscillation:

$$\ddot{x}_0(t) = \begin{cases} 0 & \text{if } \ t \leq 0, \\ a_0 \sin(\Omega t) & \text{if } \ 0 < t \leq 2\pi N/\Omega, \\ 0 & \text{if } \ t > 2\pi N/\Omega. \end{cases}$$

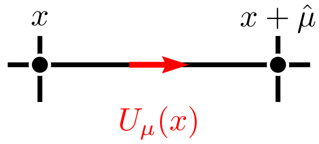
Interpret the results. What happens when $\Omega = \omega$? Discuss the limiting case of $N \to \infty$.

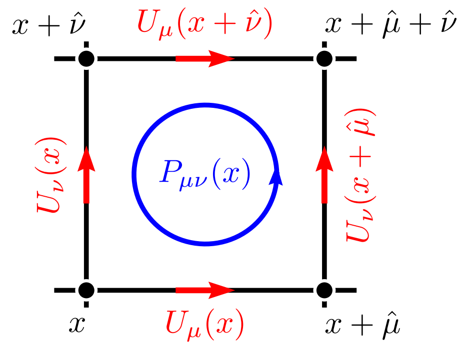
(Róbert Németh)

22. We want to describe our world with gauge theories. One way to achieve this is through the path integral formulation of the given theory, although these integrals are infinite-dimensional. However, the method of lattice field theory, with the introduction of a finite spacetime lattice Λ, makes these integrals well-defined and numerically manageable.

In gauge theories the integration variables are the gauge fields $U_\mu(x)$ (e.g. the photon of quantum electrodynamics) defined in every spacetime point x. In lattice field theory the fields $U_\mu(x)$ live on the links of the lattice, and can take on values from the gauge group of the theory. For illustration see the left side of the figure below. The matter fields (e.g. the electron) live on the sites of lattice, but such fields will not be of concern for us now.

The gauge group of QED is the Lie group U(1). It is the group of pure phases, i.e. $U_\mu(x) = e^{i\varphi} \in \mathrm{U}(1)$ with φ being real in the interval $[0, 2\pi)$. Let us discretize U(1) and substitute it with the group Z(N). The group elements are $U_\mu(x) = e^{2\pi i n/N} \in \mathrm{Z}(N)$ where $n = 0, 1, \ldots, N-1$. Note that $\mathrm{Z}(N \to \infty) = \mathrm{U}(1)$.





The Euclidean action in the Z(N) theory reads

$$S[\{U\}] = -\beta \sum_{\substack{x\in\Lambda \\ \mu<\nu}} \mathrm{ReTr}[P_{\mu\nu}(x)] \ ,$$

where $\beta > 0$ is the gauge coupling, the spacetime indices $\mu, \nu = 1, 2, 3, 4$, and

$$P_{\mu\nu}(x) = U_\mu(x)U_\nu(x+\hat{\mu})U_\mu(x+\hat{\nu})^* U_\nu(x)^*$$

is the so called plaquette (where $U_{-\mu}(x) = U_\mu(x-\hat{\mu})^*$), which is the smallest closed loop in the lattice: see the right side of the figure above. The "hat" denotes the unit translation in a given spacetime direction, while the "star" denotes complex conjugation. The vector x contains the coordinates of lattice sites: $x = (x_1, x_2, x_3, x_4)$.

a) Implement a computer code in your favorite programming language that simulates the pure Z(N) gauge theory in 4D with a lattice with M^4 sites. The simulation should use the Metropolis–Hastings algorithm. The probability density function is $\propto e^{-S[\{U\}]}$. Set periodic boundary conditions at the edges of the lattice. The input parameters of the simulation shall be M, N and β.

b) In every 10th simulation step, measure the average plaquette

$$\langle P \rangle = 1 - \frac{1}{6M^4} \sum_{\substack{x\in\Lambda \\ \mu<\nu}} \mathrm{ReTr}[P_{\mu\nu}(x)] \ .$$

Let $N \in [3, 10]$ every for every N $\beta \in [0.5, 4.5]$, and $M = 4$ (if the CPU of your computer is good enough you can try larger values of M too). Find a fine enough resolution in β. Compute the expectation value $\langle P \rangle$ as an average over your measurements. Start the simulation from a so-called cold start, i.e. every link is set to $U_\mu(x) = 1$ ($n = 0$). Note that the simulated system must first thermalize, i.e. find its equilibrium, and so these initial configurations shall be left out of the average when computing $\langle P \rangle$.

c) In the pure Z(N) gauge theory there exists a threshold $N = K$, below which only two, and above which three phases of the system can be distinguished as β is varied. E.g. the phase transitions are marked by "jumps" along the curve $\langle P \rangle(\beta)$. Naturally, these rapid changes will only be real jumps in the thermodynamic limit $M \to \infty$. What is the threshold value K?

(Dávid Pesznyák)

23. In a recent article, Kondor and Papp (https://arxiv.org/abs/2412.13580) has found that the first and second excited states of an 8 spin Sherrington–Kirkpatrick spin glass model are positioned on the surface of a torus (Fig. 10 in the article). Ignoring the energy values (not requiring that they are first or second excited states), determine if torus topologies with other size are present in this system or not. Generalize the result for an N spin system and provide an algorithm that find the tori.

(István Csabai)

24. We investigate the entanglement patterns of multiple qubits, and the unitarity properties of certain associated linear operators. We will use the von Neumann entropy as a measure of entanglement. If a quantum system is divided into two parts, A and B, then the entanglement between the two sub-systems is defined as

$$S_{A,B} = -\mathrm{Tr}\Big(\hat{\rho}_A \log(\hat{\rho}_A)\Big),$$

where $\hat{\rho}_A = \mathrm{Tr}_B \hat{\rho}$ is the so-called reduced density matrix of subsystem A, defined via the partial trace over the degrees of freedom in subsystem B, and $\hat{\rho} = |\Psi\rangle\langle\Psi|$ is the full density matrix.

We start with the case of $N = 2$ qubits. For each qubit we will use the standard basis vectors $|0\rangle$ and $|1\rangle$. Every state of the two-qubit system is described by a vector $|\Psi\rangle \in \mathbb{C}^2 \otimes \mathbb{C}^2$, which has components Ψ_{ab} in the given basis, with $a, b = 0, 1$. For each vector we construct a linear operator $\hat{A} : \mathbb{C}^2 \to \mathbb{C}^2$ in the given basis via the identification

$$A_a^b = \Psi_{ab}$$

a) What is the maximal entanglement between two qubits, as measured by the von Neumann entropy? For which states is this maximal value reached? What is the form of the reduced density matrices?

b) Show that for these states the resulting linear operator $\hat{A}$ is *proportional* to a unitary operator.

Let us also consider the case of $N = 4$ qubits. The states are given by elements of the 4-fold tensor product space, and they are described by components Ψ_{abcd} in the given basis. We can divide 4 qubits into 2 pairs in three different ways. For each such bi-partition we compute the von Neumann entropies.

c) What is the maximal entanglement between two pairs of qubits? What is the form of reduced density matrices in these cases? Show examples for the states.

d) Construct three linear operators $\hat{A} : \mathbb{C}^2 \otimes \mathbb{C}^2 \to \mathbb{C}^2 \otimes \mathbb{C}^2$ for these three bi-partitions. Show that if the entanglement is maximal for a selected bi-partition, then the resulting linear operator is proportional to a unitary operator.

e) We have a total number of 3 different bi-partitions. Is it possible to have maximal entanglement for two different bi-partitions *at the same time?*

f) Is it possible to have maximal entanglement for all three different bi-partitions *at the same time?* Perform numerical searches with random vectors.

(Balázs Pozsgay)

25. When Dr. Zero Absoluto, the eternal president of Goomy Beach, the Father of the Homeland, etc. learned that astronomers discovered a habitable, and even so, inhabited planet (named Zeta Monoculi) orbiting around a star not far from our Sun, he immediately summoned his court chief physicist and the president of the Goomy Beach Football Association.

"What do you already know about the inhabitants of that planet? Do they have legs? Two ones?"

"That planet is almost the twin of Earth! It has the same size, same atmosphere, temperature, and surface gravity—so we can assume that its inhabitants are similar to us."

"Well, if they are so wonderfully similar to us, they must know how to play soccer. And then we have a duty to play against them, proving the superiority of football of our country. Get to work right away!"

And they did so. The chief sportsman began selecting the members of the national team and developing the physical endurance of the players so that they would be in outstanding shape even after many years of space travel.

At the same time the chief court physicist, dr Aly Tudde Mynek, started designing a photon rocket. The photon rocket is known to emit photons, that is, zero-mass particles moving at the speed of light, thereby providing the necessary thrust for acceleration. The photons are obtained by transforming the material of the asteroid carried by the ship (the details are industry secret, but in any case the conversion process is 100% efficient.) The rocket operates in "candle" mode, consuming its fuel evenly, thus the mass of the spacecraft reduces from the initial M_0, after proper time T, to the value of the payload mass m. Even though m contains the soccer players, their food, shoes, even minor gifts, it is still negligibly small compared to the initial mass of the ship. The payload was later supplemented with a few soccer balls, because they did not trust the quality of the balls produced by the galactic industry—but this did not change the mass ratios much.

The parameters of the photon thruster were precisely adjusted so that the fuel would run out just as the ship arrived at the target planet.

They had almost started building the spaceship when one of the soccer players, who read a lot of science fiction, asked the coach how they were supposed to get out of the near-light-speed spaceship as it passed the Zeta Monoculi.

The chief physicist scratched his head. "I ain't no football guy," he apologized, and started revising the itinerary anyway. According to the new plan, the ship accelerates at the beginning of the journey, flips around at a well-chosen spot, and uses its engine to brake for the rest of the journey. The "candle" mode remained, certainly, since this is how the engine was built. However, the number of photons emitted over unit time had to be recalibrated. The rocket's data has been modified so that according to the new itinerary, the fuel runs out just as they reach the Zéta Monoculi. (They didn't reserve any fuel for the players to return home. The dictator explained to them that after the match—they should achieve a landslide triumph—they should stay on Zeta for a while, and then, mastering space engineering and astronomy, build a new ship and look for another inhabited planet. They should ake the news of the Goomy Beach football there too. Then even further—the limit is the sky with stars and black holes!)

The moment of the grand departure was approaching. Then the overconcerned player raised the question: what happens if the aliens do not know the rules of soccer on Earth. So the preparation and the whole trip could be in vain? The chief physicist, who had seen a lot, quickly found the solution. He prepared a terse but pithy radio message that, in addition to a polite invitation to the game also included a description of the rules of football and a call for the Zeta football team to begin training immediately after the message was received. This message was broadcast to the Zeta Monoculi by the radio telescope made from the leaves of the rubber palm trees at the moment of the photon rocket's launch.

During the ceremonial farewell party for the team leaving on the galactic mission, the dictator expressed his request to the players to spend the entire journey with training. (For this purpose, a full scale soccer field was also built into the spaceship at the last moment. Such an order has never caused problems at Goomy Beach. Not even now.)

The coach, sensitive to fair play, had previously asked the physicist to make sure that the training time of the Goomy-beach team training in the spaceship, and the Zeta team preparing on their own planet were the same when programming the itinerary. Ali Tudde Mynek shook his head and replied:

“Unfortunately, this is not possible.”

“Yes, I’ve heard about the time distortion and the twin paradox, too,” the coach expressed his sadness.

But he soon found an article that discussed training methods in unusual gravity conditions. It turned out that the training efficiency, “fitness rate”, i.e. the increase in fitness per time unit, is inversely proportional to the local gravitational acceleration. The article introduced the concept of “effective fitness”, which is proportional to the time integral of the previously defined “fitness rate”. The coach came up with the question again, asking the physicist that the effective fitness of the two teams shall equal when the ship arrived. This request—after considerable amount of calculation—was deemed feasible by the presidential physicist.

“Fair play is before all else! With the ideas of Absuluto Zero in our hearts, the Goomy-beach team will crush the weakling Monoculars even under equal conditions and equally effective training!” reported the coach to the dictator. Then he turned to the physicist:

“I hope you programmed that rocket properly! No matter how sturdy my kids are, they would only be able to withstand 10,000 g for a single moment!”

“No need to worry, I met your conditions exactly,” he said.

Our question is simple: how far is the Zeta Monoculi from Earth?

We ask for a numerical answer with two decimal places.

Hint: the time and distance units year and light-year are used during the calculation. In addition, it is advisable to use the approximation 1 light-year/(1 year)$^2 = 1\,g$, where g means the gravitational acceleration on Earth.

(Gyula Dávid)

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