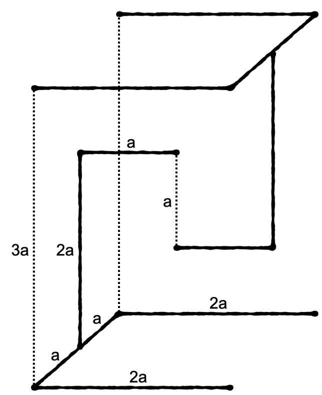
PROBLEMS OF THE 53rd—25th INTERNATIONAL—RUDOLF ORTVAY PROBLEM SOLVING CONTEST IN PHYSICS 17 February—27 February 2023

1. A table was constructed according to the *tensegrity* principle, with the outline shown in the figure below. The object consists of two identical, homogeneous mass density (along the rods) rigid frame (thick lines), and three ropes connecting these (dashed lines). The bottom base of the table is a square with $2a$ sides (with one edge missing). The top plate lies vertically above the bottom square. The height of the table—that is, the length of the longer rope—is $3a$, whereas the shorter rope is a long. The frame corners are all at right angles.



Lets assume that the base of the tensegrity structure is fixed on a horizontal plane. Which is the direction of the displacement of the top corners if we happen to cut

a) the shorter, b) one longer rope?

Give the displacement directional vector of each corner.

Here is a video about the construction of a *tensegrity* table: https://youtu.be/R0nxjj5jPDs

Further informations: https://en.wikipedia.org/wiki/Tensegrity

(Előd Gáspár Merse)

2. Consider the following simple model for a battle between two armies: Within a unit of time, each soldier of army A shoots α enemy soldiers, while each soldier of army B shoots β enemy soldiers. The battle ends when one of the armies is eliminated.

How does the outcome of the battle depend on the parameters α and β, and on the initial army sizes? How long does the battle last? How many soldiers of the victorious army survive the battle?

Using the simple model, possibly with small extensions, analyse the following tactical/strategic concepts: offensive power, defensive power, surprise attack, concentration of force.

(Zsolt Bihary)

3. We made a measurement with a solar panel in Budapest (Hungary) on a perfectly clear day (no clouds) on the 19th of June.

   We measured the power output of the panel (see data below). The temperature of the ambient air was $T_a = 29.1 - 0.16\,(t - 14.6)^2$ [C] where t is the local time (measured in hours), i.e. the hottest temperature was measured at 14:36 local time.

   The solar panel was of course warmer than the air, and had a temperature of $T_p = T_a + K\,P$, where P is the power output of the solar panel and K is a constant. The efficiency of the solar panel (to convert solar energy to electricity) is decreasing with increasing temperature by $0.35\%\,/\mathrm{C}$. The solar panel is tilted by 4 degrees with respect to the horizontal plane (i.e. almost horizontal), and oriented at 184 degrees (i.e. almost south, but 4 degrees off to the west).

   Questions: What was the fraction of solar energy that is absorbed or reflected by the atmosphere at solar noon?; at 8:00 in the morning?; and at 19:00 in the evening?

   And what is the value of the constant K?

   The measured instantaneous power output (in kW) is given below, for every 10 minutes, starting at 7:20 in the morning till 19:10 in the evening:

   0.81, 0.90, 0.98, 1.07, 1.16, 1.23, 1.32, 1.40, 1.47, 1.55, 1.63,
   1.70, 1.76, 1.83, 1.89, 1.94, 2.00, 2.04, 2.10, 2.15, 2.17, 2.23, 2.28,
   2.29, 2.31, 2.34, 2.36, 2.35, 2.38, 2.41, 2.40, 2.40, 2.43, 2.39, 2.42,
   2.44, 2.41, 2.41, 2.37, 2.34, 2.33, 2.30, 2.28, 2.24, 2.21, 2.17, 2.12,
   2.07, 2.02, 1.98, 1.91, 1.87, 1.81, 1.75, 1.68, 1.61, 1.54, 1.46, 1.39,
   1.32, 1.24, 1.16, 1.07, 0.99, 0.90, 0.82, 0.73, 0.65, 0.55, 0.47, 0.39, 0.31.

   (Gábor Veres)

4. In a meat shop a total of M kg of meat was sold in N portions in a day. Each portion was wrapped in a piece of paper and measured on a scale. According to consumer protection regulations, before a piece of meat is measured, the wrapping paper must be placed on the scale and the reading of the scale has to be reset to zero.

   Let's assume that the random measurement error of the scale in repeated measurements follows a normal distribution

   a) with a constant standard deviation of σ, regardless of the mass measured, or

   b) with a standard deviation proportional to mass, where the value of σ is increasing with increasing mass measured, as a small constant fraction ε of the mass.

   The total mass of the N pieces of wrapping paper used that day was m kg. At the end of the day, what can we say about selling m kg of wrapping paper at meat price?

   More precisely, what is the probability of selling x part of the m kg of wrapping paper at meat price assuming the type a) or type b) error of the scale as defined above? What is the probability of selling the total amount of wrapping paper at meat price?

   How would the answers change if the butcher violated the consumer protection regulations that day and did not reset the scale's reading to zero after placing the wrapping paper on it?

   Do the results depend on whether all the N pieces of meat had the same mass, M/N, or the masses of the individual pieces followed a normal distribution centered around M/N?

   Please answer the above question with a sensitivity analysis of the parameters M, N, m, σ and ε, including extreme cases.

   (Oszvald Glöckler and Gyula Dávid)

5. A ping pong ball is at rest on the top of a horizontal ping pong racket. In a given moment, we start to move the racket in a purely translational motion so that its center moves along a horizontal circle of radius r with constant speed v (in the meantime the plane of the racket is kept horizontal). Describe the motion of the ball. What is the trace left on the racket by the graphite-covered ball? (Assume that the ball does not leave the surface of the racket and it does not slip on it).

(Máté Vigh)

6. A cylindrical body of mass m and moment of inertia Θ, rolling around its horizontal axis, is gently placed on a table, without hitting it. Just when the body touches the table, it is released. The body initially rotates with angular velocity ω_0, and its horizontal initial velocity v_0 is perpendicular to the axis of the body. (The angular velocity may be either positive or negative). The coefficient of friction between the body and the table is μ, whereas the rolling drag is negligible.

   The body does not bounce off after releasing, but for a while it slips, then later rolls without friction.
   How do the final velocity V depend on the coefficient μ of friction and the parameter $k = \Theta/mR^2$ describing the mass distribution of the cylinder? Explain the result.

(Gábor Takács)

7. In the far future a global gravitational underground train system is constructed within the interior of Earth, such that straight tunnels connect the cities on surface. At the beginning of the trip, the subway trains start from zero velocity. Assume that Earth is a homogeneous sphere, and neglect any forms of friction or costs related to construction and cooling of the tunnels. However, let us do take into account the effects from the rotation of Earth.

   Calculate the travel time between any pair of points on the Earth surface (determined by geographical coordinates). How do the times of onward and return trips between two cities relate to each other? What is the arrival speed of the trains at the other end of the tunnel? From any given city, which is the destination with the shortest transit time?

(Ákos Gombkötő and Gyula Dávid)

8. A thin hoop of radius R and mass M resides in a vertical plane. A point mass m is fixed to the inner perimeter of the hoop. This system is placed onto a horizontal, initially stationary conveyor belt in its stable equilibrium position so that the plane of the hoop is parallel with the sides of the belt. In a given moment the conveyor belt starts to move with constant acceleration a.

   a) At least what should the value of a be if the point mass passes through its topmost position?

   b) Find the maximal angular speed of the hoop in the case when the acceleration of the conveyor belt is slightly smaller than the value calculated in the previous subquestion.

   The hoop does not slip on the conveyor belt. The plane of the hoop remains vertical, so the motion can be considered two-dimensional.

(Máté Vigh)

9. Turbulent Anna and Laminar Bob are making experiments in two different drop towers: they drop objects and measure their velocity and acceleration. They know that the gravity acceleration is $g = 9.81$ m/s^2. They also know that in one of the towers the air drag force is always proportional to the velocity, and in the other tower it is proportional to the velocity squared. They are not aware of the meaning of their own names, and they try to decide which law applies to which tower, but as we will see, they are not very lucky with that.

   First, they both take their own objects and drop them, wait for a long time, and they both measure the final, asymptotic velocity of their objects (i.e. when the acceleration reaches zero). We assume that the drop towers are high enough for the bodies to approach the asymptotic velocity without reaching the floor. They call each other on the phone and they find, to their surprise, that both of them got the same velocity value. Let us denote this velocity by V.

   Thus, they agree on their second experiment, which is: they both measure the acceleration of their object, precisely 1.2424 seconds after they have dropped it. They call each other and they are surprised now even more, because they got, again, the same result!

   Well, they apparently did not make much progress..., but can you, based on the above information, calculate the velocity V ?

   (Gábor Veres)

10. The three-body problem is a classical one in celestial mechanics: we study the motion of three point-like objects considering their mutual gravitational attraction. The restricted 3-body problem is a limiting case when the mass of one of the objects is much smaller than that of the other two (e.g. a spaceship in the Earth–Moon system). It is known for long that in case of two large objects orbiting around each other, there are five special, so called Lagrange points in the orbital plane, where a third, small body can permanently dwell, and the complete rotating configuration forms a stable shape. Three out of the five points are falling on the straight line connecting the large objects, whereas in case of the other two, the three masses form a regular triangle.

    Let us study the electric analogue of the restricted 3-body problem. Assume the two larger bodies (with masses M and m) have electric charges of equal magnitude but opposite sign ($+Q$ and $-Q$). These two objects orbit around each other with angular velocity Ω on a circular orbit due to the attractive force. (Let us neglect other forces, as well as emission of electromagnetic waves).

    Assume that a third, smaller body of mass μ is moving in the plane of orbit of the two larger objects, with electric forces acting on it due to its charge q (where $\mu \ll m, M$ and $|q| \ll |Q|$, note that q may be either positive or negative). Let us study if there is such a point along the line connecting the two larger bodies where the smaller object remains stationary relative to those—that is when the whole configuration rotates with the angular velocity Ω.

    How many such points are there, and how does their number depend on the parameters of the problem? Test the stability of the equilibrium points as well.

    Are there any other Lagrange points besides those falling on the line connecting the two large bodies?

    (Gyula Dávid)

11. The question of a potential time-dependence of the gravitational constant G was raised by Dirac. Measurements indicate that this change should be very weak. Let us assume nevertheless that this is not necessarily the case. Investigate numerically how do the Keplerian orbit of Earth, and the related phase space structures change if $G(t) = G_0(1 + \alpha\, t)$, and $|\alpha| \approx 1/\text{year}$, or if $|\alpha| \ll 1/\text{year}$. In the latter case, how well the numerical calculation approaches the results following from the theory of adiabatic invariants?

    (Dániel Jánosi and Tamás Tél)

12. Dyons are hypothetical particles, which are not only electrically, but also magnetically charged. The latter is a source of a radial magnetic field $\mathbf{B}$, just like the electric charge is the point-like source of a radial electric field $\mathbf{E}$.

Hint: lets work in CGS units. In this case, the units of electric and magnetic charges are the same, just like that of the fields $\mathbf{E}$ *and* $\mathbf{B}$.

a) Fix a dyon with electric charge Q and magnetic charge g in the center of a coordinate system. In the resulting field, a small particle of mass m, zero magnetic charge and electric charge $-e$ moves (that is, an electron). Neglect the effects of electromagnetic radiation emission, as well as retardation effect due to the finite propagation speed of the interactions.

Try to find a possible motion when the electron revolves on a circular orbit of radius R, with a constant angular velocity. How should one choose the signs of the fixed dyon charges, to make this possible? What is the distance r of the electron from the origin (the dyon)? Calculate and plot the function $r(R)$. Is there an upper or lower bound for the possible values of r and R? What is the speed of the electron along its trajectory? What is the orbital period T? Let us try to find a rule analogous to Kepler's third law.

b) Extra problem for gourmets (the complete answer to part a) claims maximal score without dealing with the extra problem): Study the problem in the frame of the special relativity theory. (The fields generated by the electric and magnetic charges are the same as in the non-relativistic case, the emission of electromagnetic waves and retardation effect can be neglected as well.) Study the questions of the part a) again, give the answers and analyze the distinctions.

(Sándor Lengőy)

13. FlatEarth is planet with the shape of a square based prism. The sides of the square side, which they call the 'Perimeter', are 10000 km long, whereas the thickness of the prism is 2000 km. The mass distribution of the planet can be assumed homogeneous, and its density is the same as the average density of our Earth. Most inhabitants live in CenterCity, close to the middle of the Square. Some of their earlier adventures were discussed in 2020 during the course of the Ortvay Competition (see Problem 27), which revealed the affection of FlatEarthians to winter sports—this is obvious being the complete planet covered with frictionless ice.

With the improvement in alpine techniques, the bravest sportsmen could get further and further from CenterCity, and closer to the Perimeter, and even to the vertex, beyond which there is only the outer space. On the way back, they exploited the special gravitational field of the planet, and slid back on frictionless skis. Certainly—beyond skis and climbing iron—they needed spacesuits as well. These are streamlined high-tech items: we can neglect both friction and atmospheric drag.

And there came the magnificent moment, when the two bravest mountaineers, Fred and Barney, stood for the first time on the tip of the planet—the vertex of the square side—with only the heavens above. At their feet, the three edges of the prism run in three perpendicular directions. They complete the obligations—signing the peak book, taking selfies—then it is time to go onwards. The question is, where?

Fred prefers returning to CentralCity, as he knows the celebration at this favourite pub is an important part of the mission. Barney however, considers that once he got so far, it would be a sin to waste the acquired potential energy. Why not visiting the adjacent vertex of the Square? All he needs to do is to slide along the relevant edge of the Perimeter, starting right here at his feet.

They say goodbye, and both start simultaneously, setting their skis in the corresponding directions, with zero initial velocity: Fred towards the center, Barney towards the adjacent vertex along the Perimeter.

How much time is needed for Fred, and for Barney, to complete this part of the journey, respectively? (For some reason, the inhabitants of FlatEarth measure the time in units of 'clicks', where one 'click' corresponds to the Equatorial low earth orbit time of satellites in our own Earth...) Please give a numerical answer, using the unit 'click'.

(József Cserti)

14. In my favourite YouTube channel: https://www.youtube.com/@TrainExperiments they place all kinds of *stuff* under the wheel of a (slowly moving) locomotive.

a) Some *stuff* is simply pushed away by the wheel.
b) Some *stuff* is projected by the wheel rather than being rolled on.
c) Some *stuff* is rolled on then rolled off peacefully.
d) Some *stuff* is rolled on then projected backwards.
e) Some *stuff* is rolled on but then *stuff* blocks the rotation of the wheel which instead slips on named *stuff*.

Under adequate simplifying assumptions, characterise the properties of *stuff* that are subject to the respective cases above. How does the situation change if the locomotive moves fast?

(Márton Balázs)

15. We need to manufacture some type of product from zero articles to a target number X of articles in time T. For convenience we use a continuous variable to measure production, so let $x(t)$ be the number of items produced until time t, with $x(0) = 0$, $x(T) = X$. The cost of augmenting the number of products by dx amounts to $a\,\dot{x}(t)\,dx$. In the time interval dt, we need to add to this the cost of storage of the items produced up to that point, given by $b\,x(t)\,dt$ (a and b are positive constants).

a) What $x(t)$ production curve, with an arbitrary, but fixed, T, minimizes the total cost

$$S = \int_0^T L(x, \dot{x})\,dt,$$

where $L = a\dot{x}^2 + bx$? Determine the value of the minimal cost as a function of X, T.

b) Solve the above problem with undetermined T, i.e., minimize the total cost in terms of T as well. Calculate the minimal production cost as a function of X, and present the corresponding $x(t)$ production curve.

c) Let the selling price for a unit of product be p. We can assume that all products manufactured can be sold. In the absence of any limitation on T, by what total number of products X would be the profit, that is, the difference between the sales revenue and the production cost, be maximal? In plain English, solve this variational problem with free endpoint.

Are the optimal $x(t)$ solutions for cases a), b) and c) unique?

Pay attention to the fact that the number of articles $x(t)$ can neither decrease ($\dot{x} \geq 0$), nor can be negative ($x \geq 0$). As a further reminder, stationarity alone does not solve the problem without verification that it indeed yields a globally minimal cost. We expect theoretical solutions, which may be graphically illustrated.

(Géza Györgyi and Vázsony Varga)

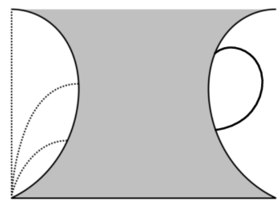
16. Consider a thin elastic band, with cross-section of small square with sides a, which is being held with mechanical tension T. We can categorize the shape changes of the band with twists (illustration on the left) and curls (illustration on the right).

According to our experiences (try it!), if a thin rubber band is twisted enough times, curling will occur, but if we increase the tension, the curled state will realign into a twisted state.

Explain the phenomena, and give an estimation to the minimal number of twists which allows the formation of a curl.

(Ákos Gombkötő)

17. Consider a thin walled rotationally symmetric beer mug, its bottom plate is a circle of radius R. Its notable property is that wherever we make a small hole on the sidewall, the initially horizontal stream of liquid falls precisely to the edge of the bottom plate of the object. Furthermore, it is known that seen from above, using any oblique line of sight, one can observe totally only half the area of the bottom plate.



a) How much time it takes until the full glass of beer empties, if we punch a small $r \ll R$ hole on the bottom?

b) If we accidentally flip the mug, and it lays sideways on a horizontal plate, at most how much beer can remain inside?

(Előd Gáspár Merse)

18. The simplest example of the capillary effect occurs when a thin, cylindrical shaped, vertical tube with the length of H and with both ends open is dipped into a liquid. The level of liquid may rise inside the tube due to surface tension assuming that the contact angle is less than $90°$. The radius of the cylinder is r_0. The rise of the liquid can be estimated by using Jurin's law. Let us denote this height of rise with h_J.

In this exercise, the modified version of Jurin's law has to be derived in two cases. In both cases, we neglect effects stemming from the curvature of meniscus, similarly to conventional Jurin's law.

a) Let us investigate the case when the upper end of the tube is closed. The tube is positioned such that the lower, open end just touches the liquid surface (no actual dipping). The air inside the tube can be modeled as an ideal gas with constant temperature. At the initial time, when the lower end touches the liquid surface, the pressure of the internal air equals the external pressure p_0. The density of the liquid is denoted by ρ and the gravitational acceleration is g. How does Jurin's law change for a tube with closed upper end? How much is the rise if $\rho g h_J < p_0$?

b) In the second case, both ends of the tube are open, but the cross-sectional radius slightly decreases with altitude as $r(z) = r_0 - \lambda z$. Here, the axis z is perpendicular to the free liquid-gas contact surface and $z = 0$ indicates this surface. How does Jurin's law change for the narrowing tube?

c) Estimate the elevation numerically using the data below in three cases: 1) open, straight tube—the original form of Jurin's law; 2) straight tube with closed upper end—use the formula derived for the part a) of the problem; 3) open upper end, but narrowing tube—use the formula derived for the part b) of the problem. Data: $r_0 = 1\,\mathrm{mm}$, $H = 5\,\mathrm{cm}$, $\rho = 1000\,\mathrm{kg/m^3}$, $\gamma = 0.072\,\mathrm{N/m}$, $g = 9.81\,\mathrm{m/s^2}$, $\Theta = 52°$, $p_0 = 1\,\mathrm{bar}$, $\lambda = 0.016$.

(Ádám Bácsi)

19. It is a well known empirical fact that we can easily spill coffee from a mug while walking if the eigenfrequency of sloshing in the vessel encounters a resonance with the frequency of our steps. Give an estimate of the sloshing eigenfrequencies for a two-layered cocktail in a cylindrical mug, consisting of stably stratified fluid layers of different densities, separated by a sharp interface, as a function of the densities and thicknesses of the two fluid components.

(Miklós Vincze)

20. A reservoir with volume V contains two different kinds of ideal gases: A and B. Initially both of them are in gas phase, but at lower temperature they can undergo phase transition.

Far from the critical point the phase coexistence curve for substance A is described by the Clausius–Clapeyron equation:

$$\frac{\mathrm{d}p_A}{\mathrm{d}T} = \frac{p_A L_A}{RT^2},$$

where R is the universal gas constant, p_A is the partial pressure of gas A and L_A is the specific latent heat for gas A, which is constant in the parameter region studied. The equation is similar for substance B. Latent heats L_A and L_B are different. The volume of the liquid is negligible compared to the volume of gas phase. We can assume that the liquids form an ideal solution.

How does the pressure in the reservoir change as the temperature decreases? Plot the curve.

(Máté Veszeli)

21. Consider a body of finite size floating weightlessly in a gaseous media. From the pure mechanical point of view, it can be modelled as a point mass. For the body, we define internal energy U through the expression

$$U = E - m\,\dot{\mathbf{r}}^2/2,$$

that is, with the difference between total energy E and mechanical energy.

The equation of motion for the body is

$$\frac{d}{dt}(m\,\dot{\mathbf{r}}) = \mathbf{F} + \mathbf{D},$$

where $\mathbf{F}$ denotes external force, $\mathbf{D}$ denotes dissipative force.

We can assume that the dissipative force does not modify the total energy of the system, which can only be changed by the work done by the external force. The temperature of the system is positive. The heat conduction within the body is fast enough for the temperature distribution to be considered homogeneous, while the heat capacity is large enough for the temperature to not change significantly during the process.

Our question: If the entropy of the system can not decrease, then what inequality can be written for the dissipative force? Interpret the result.

(Ákos Gombkötő, Péter Ván)

22. There is a spherically symmetric atmosphere around a spherical planet. The refractive index $n(r)$ of the atmosphere depends only on the distance from the surface of the planet, and monotonously decreases from the surface value n_0 to the value $n = 1$ infinitely far in the vacuum of space.

a) An astronaut, standing on the surface, projects a horizontal beam of light from his handheld torchlight (that is, the light beam is initially parallel with the surface). Determine the necessary conditions allowing the beam of light to reach to the outer space—or alternatively, that the light would 'fall' back on the surface. Disregard effects due to the rotation of the planet, or any attenuation or scattering of the light beam.

b) A spaceship approaches the planet from infinitely far. Due to the curved light path in the atmosphere, the solid body of the planet appears to be larger than it is actually, when observed by the astronauts. Determine the apparent linear magnification. How this question is related to the previous one? *(Hint: consider turning the light rays around!)*

(Gyula Dávid)

23. We have an ohmic resistor network, which consists of six nodes. We measured the resistance R_{ij} between each pair of nodes i and j (measured in unit Ω). The results are collected in the following matrix $\mathbf{R}$:

$$\mathbf{R} = \begin{pmatrix} 0 & 13 & 13 & 24 & 19 & 40 \\ 13 & 0 & 10 & 13 & 10 & 31 \\ 13 & 10 & 0 & 19 & 12 & 33 \\ 24 & 13 & 19 & 0 & 13 & 34 \\ 19 & 10 & 12 & 13 & 0 & 21 \\ 40 & 31 & 33 & 34 & 21 & 0 \end{pmatrix}.$$

Using this data, please, reconstruct the adjacency matrix of the network and the resistances of the resistors connecting the nodes.

(*Hint: do not use decimal fractions in the calculation and in the result.*)

(Gábor Vattay)

24. A point-like dipole of mass M and magnetic dipole moment $\boldsymbol{m}$ can move without friction in the plane of an equilateral triangular wire carrying a current I. The direction of the dipole moment is perpendicular to the plane of the triangle. What is the magnetic dipole's period of oscillation around the triangle's center for small displacements? What is the condition for stability?

(József Cserti)

25. Consider a long cylindrical permanent magnet of radius r, which is placed on an infinitely large table made of ferromagnetic material. The base of the cylinder is in contact with the smooth surface of the table. The magnet has a constant and homogeneous axial magnetization $\mathbf{m}$. Consider the ferromagnetic material as a medium with a constant but very high relative permeability. What is the magnetic force acting on the magnet?

(Gábor Széchenyi)

26. Two circular conducting wire loops of radius a are placed over a cylinder made of non-conducting material with a radius slightly smaller than a and standing in a vertical gravitational field with a constant acceleration g. The first ring is fixed, while the second ring, whose resistance is R, can move along the cylinder without friction. Initially, the second ring is located at a certain height above the first one and it is at rest.

A constant current I flows in the first ring driven by a current generator. If the second ring is dropped, an eddy current is created in the ring as the magnetic flux through the ring changes. Due to Lenz's law, a braking force acts on the second ring.

Calculate the exact braking force and the equation of motion of the ring.

Using the magnetic dipole moment approximation of the magnetic field of the first ring, calculate the braking force and the equation of motion in the limiting case when the distance between the two rings is much larger than a.

Show that the force obtained in this way is the same as the result obtained based on the corresponding power series of the exact expression of the force.

Analyze analytically the long-term behavior of the movement under different initial conditions.

The feedback of the second ring on the first ring is negligible due to the current generator drive of the first ring.

(József Cserti)

27. As it is commonly known (see *The Cyberiad* from Stanislaw Lem), the home planet of the beings at the Highest Possible Level of Development—HPLDs—is cube shaped. Possessing high technological levels, they could make a cubic planet—then, just for the fun of it, why not?

Certainly, a home planet worth its name must be protected from cosmic influences with a magnetic field. In absence of the natural magnetic field of the rotating iron core of the planet, the HPLDs solved this such that they implemented a high power current source to the middle of the cube. The current was guided along one of the axial diagonals to two opposite vertices of the cube, from where (symmetrical thus identical) conductive rails fixed on every edges of the cube closed the circuit. The magnetic field thus created by the circuit gave a reliable protection for the hard working HPLDs from the external effects for millions of years, both on the surface, and inside the cube.

Recently however, the (solar) winds of change blow through the Galaxy. More and more criticism rose from those beings living common lifes on commonly spherical planets, addressing the open perkiness of the HPLD-s, one of the sign being the cubic planet itself. At some point the HPLD-s decided to put an end to the stream of unfounded blames, and disguise their hightech cube to look like a totally common planet. From the adjacent solar system they ferried many freightships loaded by electrically insulating clay, sand and other low-grade stones, and using this as landfill, turned their cubic planet spherical (with a thin coverage over the corners of the cube as well). From the outside, it totally looked like an utterly common CW2-class rocky, dusty, deserted spherical planet. The HPLDs were satisfied to retreat down to their cube below the rock layer cover, and continued with their activities, which to all the less developed sentient beings are anyway totally uncomprehensible.

When Trurl and Klapautius visited again the HPLDs' planet, they could not even recognise it. 'But it should be surely here!' scratched their head. 'This can not be that planet, must have been swapped,' Trurl said.

Klapautius however, took his old, reliable magnetic compass, and carefully scouted the barren, deserted surface of the planet, and mapped both the vertical and the horizontal components of the magnetic field, for the latter also the North–South, and East–West projections. He plotted these on level contour maps. When ready, he showed this victoriously to Trurl: 'See I told you so! The cube of the HPLDs hides down below the surface!'

At this point a stray comet ripped the maps, prepared with exhausting work, out of his hands and was lost forever.

This task remains to be completed by the participants of the Ortvay Competition: reconstruct the sadly lost maps which Klapautius prepared. Besides the description of the method and calculation, three well prepared maps are needed to show the components of the magnetic field as a function of the polar coordinates, and from which the existence of the underlying cubic structure can be verified.

(József Cserti and Gyula Dávid)

28. Study the motion of a mass point in a relativistic scalar field $\Phi(t, x)$. The dimension of space-time is (1+1). The action integral governing the motion is

$$S = \int d\tau \, [-m\, c^2 - \gamma\, \Phi(t, x)],$$

where τ is the proper time measured according to the world line of the particle, γ is the coupling constant between the particle and the field, m is a parameter of mass dimension and c is a parameter of space-time (its trivial name is the velocity of light).

Derive the formula determining the derivative of the rapidity χ of the particle against the proper time τ in a most compact form.

(Gyula Dávid)

29. Study the vertical free fall of a relativistic mass point in a static four-scalar field $\Phi(z)$. The coupling constant between the particle and the scalar field is $\gamma = 1$. According to the similarity to the corresponding classical problem write the coordinate dependence of the scalar field in the form $\Phi(z) = m\,g\,z$ where m is the mass constant of the particle (see the former problem) and g is a constant of acceleration dimension, the value of which is equal to the terrestrial gravitational acceleration.

Let the particle start from a point of height $z = H$ with zero initial velocity. Derive the equation of motion and solve it, give the function $z(t)$ in a closed form. (Neglect the collision of the particle to the ground, consider the function $\Phi(z)$ to be linear on the whole axis z.)

What is the maximal speed of the particle during the motion?

Why is the motion of the particle so widely different comparing to the usual free fall problem in homogeneous gravity field? Give the physical reason of this difference.

Find the limiting case which leads to the formulas of the usual free fall problem.

(Gyula Dávid)

30. Captain Jan Luc Picard scratches his bald head worriedly. He has just received the subspace telegram: the High Command of the Starfleet ordered an overhead reduction.

The relativistic Starship Enterprise has recently departed for an important mission, straight towards the Epsilon quadrant. Out of the total departure mass M_0 the useful mass (people, robots, food, drink, holographic deck, tricorders, engines, commanders' armchairs, reactor, everything) is m_0. The difference is the newly developed, exceptionally efficient fuel called anameson, filling up the fuel tanks. The engine ejects the jet of anameson with a fixed velocity $u = c/n$, where $n > 1$. The value of n can not be changed, but one can throttle the engine by changing the amount of consumed anameson $\mu(\tau)\,d\tau$ during a given interval of proper time $d\tau$. As of now, on the first day of the journey, the ship provides very pleasant conditions, since the passengers feel their Earth weight mg on the deck due to acceleration. The original plan was that they accelerate up to half the way, then turn the engines forward and decelerate again with a constant proper acceleration.

However, according to the undisputable fleet command, they must use the anameson more economically. The order specifically requires that they should use during a unit of proper time only the same fraction of the remaining fuel, as they do now at the beginning of the trip. A smartass up in the HQ figured that this way, the anameson should last forever, thus never need to refill.

Captain Picard, after a sigh, enters the information to the central computing unit, which right away starts to economize.

Let us determine the speed of the spaceship as a function of proper time.

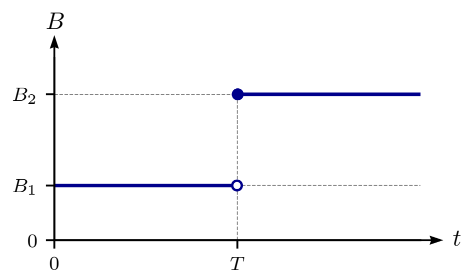
How do the apparent weight of the astronauts change during the journey? (Let us neglect the unpleasant condition, that they might need to be economical with food).

Let us estimate that relative to the original flight schedule, when does the spaceship arrive at the distant ($L \gg c^2/g$) destination.

Prepare corresponding plots showing the results.

(Gyula Dávid)

31. A spinless charged particle is confined in a two-dimensional region with a homogeneous magnetic field, which is perpendicular to the plane of motion. At an instant $t = T$, the magnitude B_1 of the magnetic field changes suddenly—that is, negligibly quickly compared to the velocity of the particle, but also slowly compared to the speed of light in vacuum—to the constant B_2. This is performed such that the initial and final magnetic fields are parallel; a possible example is illustrated in the *figure*.



a) Consider first the classical model of the problem with the particle being a point-like body. What are the general solutions of the equation of motion? Plot the trajectories corresponding to some specific initial conditions.

b) Consider next the quantum mechanical model of the problem. Initially, at $t = 0$, the system is prepared in the ground state, then at some instant $t > T$ the energy of the particle is measured. What is the probability of finding the system in the current ground state? Compute also the expected value of the measurement of energy.

Remark: For the sake of mathematical clarity, we can assume that the magnetic field is constant only in the sufficiently large area of the experiment, and decays cylindrically symmetrically outside.

(Róbert Németh)

32. Anikó and Botond are physics students, learning about quantum entanglement. They learned that in the case of two qubits the so-called Bell pair is the main example for entanglement, and that this state corresponds to the $SU(2)$ singlet. Anikó and Botond would like to understand the representation theoretic background as well. They learn that the tensor product of two fundamental representations of $SU(2)$ splits into a sum of a singlet and a triplet representation. Using the standard notations, the singlet consists of the state

$$|S\rangle = \frac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{\sqrt{2}},$$

whereas the three basis states of the triplet are

$$|+\rangle = |\uparrow\uparrow\rangle\,, \qquad |0\rangle = \frac{|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle}{\sqrt{2}}, \qquad |-\rangle = |\downarrow\downarrow\rangle\,.$$

Anikó és Botond wish to understand the entanglement of the states of the triplet representation. The vector $|0\rangle$ is interesting: it is orthogonal to the singlet, and it also has zero total magnetization.

Botond says that $|0\rangle$ has zero entanglement. The reason being that $|0\rangle$ is in the triplet representation, which includes product states, and entanglement is a quantity which is invariant with respect to rotations.

Anikó says that it is best to actually perform the computation. She says that the sign difference will not matter at all, and the entanglement in the state $|0\rangle$ (for example the von Neumann entropy) is the same as in the singlet state.

Who is right? What is the mistake of the one who is wrong?

(Balázs Pozsgay)

33. Uncle Joe lives close to the facility he calls the Large Supersonic Ambulance Car Collider. This is a circular multi-kilometer circumference test track where ambulance cars, running faster than the speed of sound, keep running around. Last year Uncle Joe was trying to spy on what was happening only relying on what he could hear through the non-transparent fence, and we had the chance to follow his experience in detail during the Ortvay Competition in 2022, problem 28. (Worth a careful reading).

This time, however, there is a big change. Uncle Joe tells his friends the latest news in the pub. It turned out that the owner of the test track, the weird billionaire (himself an enthusiastic solver of the Ortvay Competition) learned about Uncle Joe's unending interest, and invited the old fellow: why not to sit into one of the test cars? This way, he can experience first hand, with his own eyes, ears—backside even—what the ambulance cars actually do.

'Well you know when the car began to accelerate after boarding, there was a huge noise so I needed to cover my ears and close my eyes. Soon I was signaled that we reached cruising speed, so I started to listen and look around. In fact, it was a boring view. The ambulance car was running at a constant speed, therefore I only saw that the boring circular track runs towards us, and nothing happens.'

'But then I listened carefully, which gave clues on the trickery. No doubt, just as last year when I was relying on my hearing only, I could reveal the quantum teleportation and car pair production, even annihilation happening. In fact this time there was no trace of such things, but I could sense something particularly suspicious.'

'I could immediately tell: we were not the only car on the track! Ours was screeching wild, but similar sounds came from two other points on the circumference. I looked in that direction—but there was nothing. And now I know, they do not only test normal visible ambulances, but invisible ones as well!'

'Could you hear exactly *two* such ambulance cars?' the cunning student startled. 'And those two never hit you during the run?'

'Oh no, these guys are more tricky and more careful than that to happen,' Uncle Joe replied. 'The other two cars were running with just the same speed like us, therefore these were at a constant distance behind us (or before us, one can never know on these circular tracks). Therefore the sound came always from the same direction. One of the invisible cars was slightly behind us, and the other one roughly at the opposing part of the track relative to us.'

The first few questions of the problem hereby follow:

a) What is the speed of the circulating ambulance car, relative to the speed of sound (that is, Mach number)? Give a numerical value.

b) Precisely which is the direction from where Uncle Joe could hear the sound of the invisible ambulance car sirens? (Determine in terms of angular difference along the circumference).

However, Uncle Joe did not finish yet...

'I thought it is time to spill the beans. I asked the pilot to accelerate, and decelerate slightly. Sure the invisible cars need to react accordingly to avoid collision. And I listened very carefully, to observe what happens. And imagine, that right then...'

Unfortunately, Uncle Joe could not finish, because at this point the bartender closed the pub. Uncle Joe is well known for his habit of exclusively talking about adventures in the pub by a glass of wine. Consequently, both his curious audience, as well as the problem solvers of the Ortvay Competition, need to find out the answers by their own reasoning:

c) What was the experience of Uncle Joe, when the supersonic ambulance car sped up, or down, slightly?

d) We may answer, based on what was learned previously, why the cunning student in the pub was surprised about the exactly *two* invisible cars, which Uncle Joe heard. In which case would he have no reason to be surprised?

(Gyula Dávid)

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