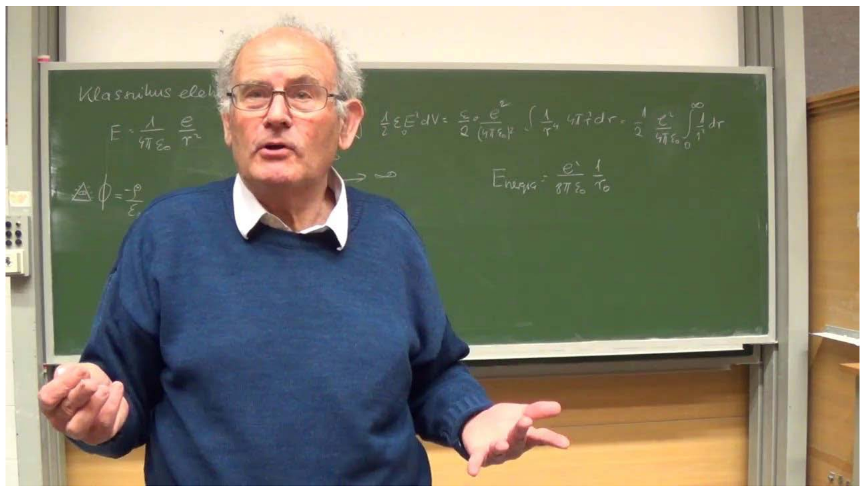
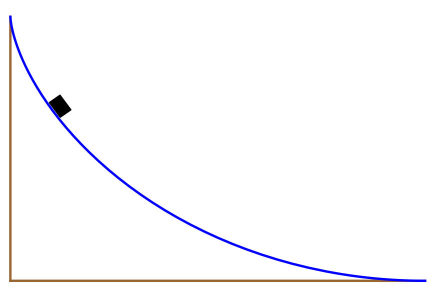
PROBLEMS OF THE
52nd—24th INTERNATIONAL—RUDOLF ORTVAY
PROBLEM SOLVING CONTEST IN PHYSICS
28 January—7 February 2022



Géza Tichy passed away in 2021, one of the founders of the Ortvay Competition,
dear teacher and good friend to many of us.
This year's Ortvay Competition is dedicated to his memory.

1. A point-like body is released without initial velocity from the top of a semi-cycloid-shaped slope. The frictional force is proportional to the normal force. The body slides down to the bottom of the slope and then stops there. What was the coefficient of friction between the slope and the body?



(Géza Tichy, 1945 – 2021)

2. Solve the equations of motion derived from the Lagrangian:

$$L(x, \dot{x}, y, \dot{y}, z, \dot{z}) = x^2 y^2 \dot{z}^2 + \frac{x^2 \dot{y}^2}{1 - y^2} + \dot{x}^2$$

Is there any physical system which can be described by these equations?

(Géza Tichy, 1945 – 2021)

3. A car with a total mass of 1200 kg travels from Neverfarm to Nowherecity, covering a distance of 100 km. The elevation of the two towns are the same on the hilly landscape, and they are connected by a straight road on the map, where there is no traffic, traffic light, or crossing. The elevation profile of the road is a *sine* function and contains precisely 100 full periods with amplitude M, where $0 < M < 100$ m. Friction and air drag acts on the car with the total force of $F = A + Bv + Cv^2$, where $A = 100$ N, $B = 5$ Ns/m and $C = 0.25$ Ns^2/m^2. The motor works at a power of $P_0 = 5$ kW even if the car stands still, and this is the necessary extra constant power during movement as well, besides moving the car. There is no regenerative breaking, thus only the break pads heat up and wear down. If the driver uses cruise control, which constant speed s/he has to set (as a function of the parameter M) in order to minimize consumption, and how much gasoline is used up then? If the driver does not use cruise control and drives optimally, what will be his/her minimal fuel consumption? (One litre of gasoline provides 9 kWh of energy).

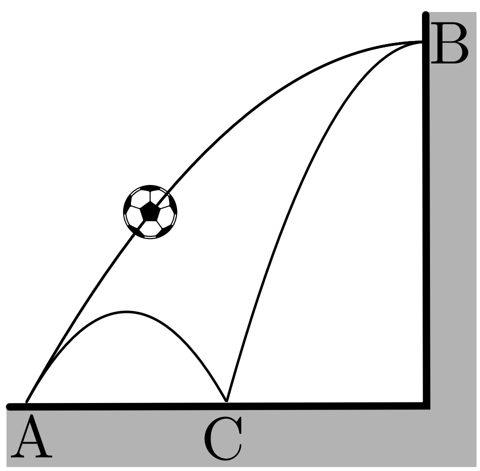
(Gábor Veres)

4. During the school holidays, we kick a football against the wall. The place of the kick is the point marked A in the diagram. The ball hits the wall exactly perpendicular at point B, and after the bounce it lands at point C. After bouncing off the ground, the ball bounces back to its original starting point. A collision with a wall or a ground can be modelled as follows: the velocity component parallel to the surface does not change, while the one perpendicular to it changes by a factor of $(-k)$. Generalize the above case and bounce the ball exactly N times on the ground before returning to its starting position.

   a) What is the value of k for a given N?

   b) How long is the ball in the air?

   Examine in detail the case $N \to \infty$.



(Zoltán Tajkov, János Koltai and Dénes Berta)

5. Investigate a mathematical pendulum whose length l is changing in time in a prescribed, uniform way: $l(t) = l_0(1 + \alpha t)$. (If one so wishes, the initial length l_0 and the gravitational acceleration g can be transformed out from the equations by choosing the appropriate time unit.) Air drag is assumed to be negligible, and the rate α is small, but not so much that the adiabatic limit is reached. Follow numerically the motions starting from different states of the phase space with special emphasis on those that switch between swinging and overturning, or vica versa. Can one find a representation particularly well suited for a global monitoring of the motions in this system?

(Dániel Jánosi and Tamás Tél)

6. A long rod is rotated around one end in a vertical plane with constant angular velocity ω_0. The gravitational acceleration is homogeneous, points downward, with magnitude g. There is a ring sliding on the rod without friction.

   The rod is horizontal at $t = 0$, the ring is at distance r_0 away from the origin and its initial velocity parallel to the rod is zero. At this moment the rod turns upward.

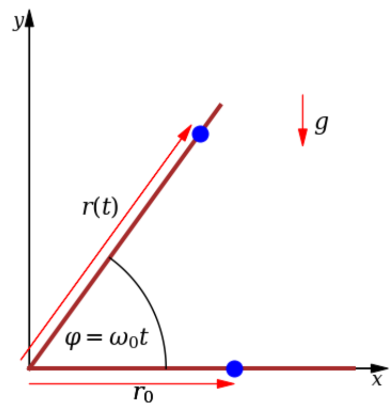
   How does the ring move? Study the ring's position when the rod becomes vertical for the first time.

   a) At what value of ω_0 will the ring be at the origin?

   b) At what value of ω_0 will the ring be at height r_0?

   c) Where will it be if ω_0 is very large?

   d) At what condition will the ring fall into the origin during the motion?



(Szilveszter Fehér)

7. Dr Ali Tudde Mynek, head physicist of Gummy Shore—based on his previous experiences—foreseen that some kind of task awaits him when Dr Absoluto Zero, Eternal President of the small equatorial country called him. The dictator moved silently around the head physicist along a constant-acceleration trajectory, until he finally presented his problem:

   'Yesterday...' he started his monologue, 'I consecrated the Port Gummy roller-coaster, named after Dr Absoluto Zero, and naturally tried it. My stomach has been jumping up and down from all the higgledy-piggledy acceleration. This is not fitting for our great nation, neither for my tummy. Your task will be to redesign the roller-coaster in a way that eliminates this uncomfortable effect!'

   Let us help the head physicist by creating the sketch of the roller-coaster! For the sake of simplicity, you may assume that the track is within a vertical plain, its shape can be described by a not-necessarily single-valued function $x(z)$ (self-intersecting is allowed). The dynamics need to be such that the car pushes the track with constant force (with C-times the rest weight of the car). Frictions are negligible. Of the possible solutions choose those which contain a 'loop', similar to those often found in roller-coasters.

(Ákos Gombkötő)

8. *This problem is dedicated to the memory of Géza Tichy (1945 – 2021), one of the founders of the Ortvay Competition, who, besides a number of other interesting and exciting concepts in physics, taught us how to treat the problems of motion with nonholonomic constraints (see Problem 8. of the Competition in 1971.)*

   A pointlike skate is sliding on an infinite flat ice sheet of inclination angle β. The mass of the skate is m, its moment of inertia is Θ. The initial velocity at time $t = 0$ is horizontal, at magnitude v_0. The initial angular velocity is ω, and in the first moments the trajectory curves uphill on the slope. The condition $v_0 > g/\omega$ holds, where g is the component of gravitational force acting in the direction of the slope.

   The movement of the skate is only allowed in the momentary direction of the blade, all movements perpendicular to this are prohibited by the constraint forces.

   The skate experiences a friction, in a direction opposite to the direction of the velocity, with a magnitude of $S = mcv$, where c is a positive friction constant with frequency dimension.

   Derive the equations of motion, and solve them analytically. Plot the skate trajectory on the slope.

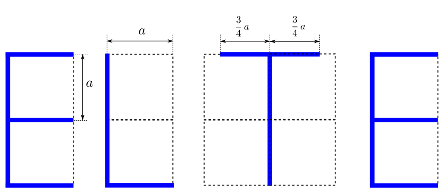
   How does the motion appear, if observed from far above? What sort of qualitative change ('phase transition') appears during the motion? What is the asymptotic solution, to which the motion converges (assuming that the slope is indeed infinite, and there is sufficient place for the asymptotic state to form)? Which are the parameters of this asymptotic state, and how do these parameters depend on the initial parameters defined above?

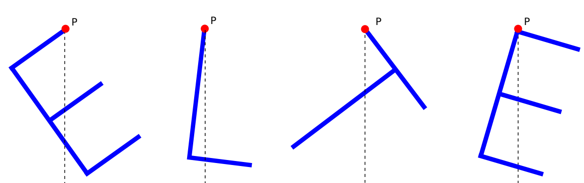
   Study the two extreme cases of the problem: a) if the friction approaches zero; and b) if the inclination angle of the slope approaches zero.

   Which are those important neglected physical conditions, which render the sliding model unrealistic? How can these inaccuracies be possibly remedied? (Only ideas are requested!)

   (Gyula Dávid)

9. The acronym ELTE stands for the native name of Eötvös Loránd Tudományegyetem—Eötvös Loránd University in English. The letters are made from thin wires with uniform mass distribution and are painted blue (see the left figure with dimensions). To dry the painted letters, we hung each one at a specific corner (at point P shown in the right figure) so that they can swing freely in all directions. What will be the period of each letter in the plane of the paper and in the direction perpendicular to it for small swings?





   (József Cserti)

10. A strange, homonuclear, triatomic molecule has the following potential:

$$V = V_0 \left[\frac{P}{L} + \frac{3\sqrt{3}}{8} \frac{L^2}{A} \right],$$

   where P is the perimeter and A is the area of the triangle, defined by the three atoms. V_0 and L are constant positive parameters.

   Determine the vibrational frequencies of this molecule, if the motion of its atoms is restricted to the plane of the triangle.

   (Máté Veszeli)

11. Based on the intensive research the physics students of the Eötvös Loránd University discovered Aliens living on a square surface of Planar Planet. This planet is a perfect, square-based rectangular prism. The side length of the square is a and the height of the prism is h. The side ratio of the prism is $h/a = 0.25$. The mass distribution of the Planar Planet is homogeneous, its mass density is the same as the average density of our Earth.

    Life is rather strange in this flatland. There is one big ocean in the middle of the Planar Planet. Along the centerline of the square, the ocean shore is at half the distance from the center to the edge of the square. How deep is the ocean in the middle of the square?

    (József Cserti)

12. It is well known that with conventional converging lenses, lens aberrations occur due to light rays running not close enough to the optical axis. This can be avoided by using a Fresnel zone plate. If the radius and the density of the rings are chosen correctly, it is possible, for example, to focus a beam parallel to the optical axis at a distance f from the plate such that the light rays near the edges of the plate also have their intensity maximum at this focal point.

    However, it is suspected that the zonal plate thus obtained will not be perfect in focusing the light from a point source placed at a finite distance t along the optical axis. Will there be points of high intensity, and where? By this, we mean those points where the intensity is infinite relative to the incoming light if the ratio of the size of the system to the wavelength tends to infinity.

    (Zoltán Kaufmann)

13. A regular triangle is bent from a uniformly charged thin insulating wire and a massive point charge is placed in the center of the triangle. Assume that the motion is confined to the plane of the triangle.

    What will be the frequency of oscillation of the charge for small displacements? What can we say if the charge's motion is allowed in all three dimensions? Is there a frame made of insulating wire from which the charge cannot 'escape'?

    (József Cserti)

14. The number of turns in an air-core solenoid with length ℓ and radius $r \ll \ell$ is denoted by N, where $N \gg 1$. Determine the self-inductance change of the solenoid if a tiny small iron sphere with volume V and relative permeability $\mu_r \gg 1$ is placed in the middle of the solenoid.

    (Gábor Széchenyi)

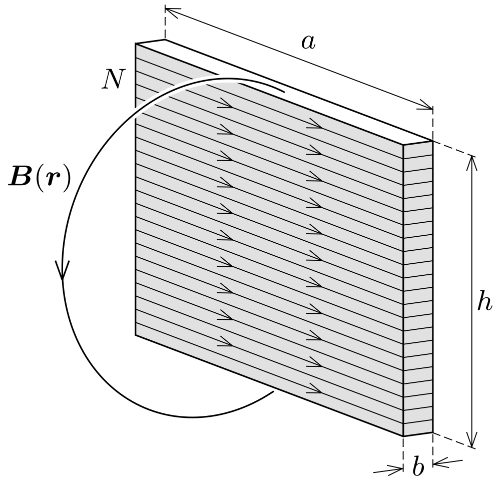
15. It is well known that due to the eddy currents induced inside the wall of a vertical tube made from a conductive non-magnetic material a strong magnet falls down slower than in free fall in a gravitational field. For a long enough tube, after some time the magnet will move at a constant velocity called terminal velocity.

    Usually, the theoretical works assume cylindrical tubes. Now, we consider a pipe with a regular N-angle base. Find the terminal velocity in such tubes for different values of N (for example, for $N = 3 \dots 8$).

    For simplicity, we model the magnet as a vertical magnetic dipole. Since the speed of the falling magnet is slower than the time scale of the decay of eddy currents, the self-induction effects can thus be ignored.

    (József Cserti)

16. A solenoid of length h has N turns. The cross-section of the solenoid is a rectangle of sides a and b, where $b \ll h \ll a$ (see the figure). Constant current I flows in the coil wire. What is the shape of the magnetic field lines lying in the plane which is perpendicular to the sides a and contains the geometrical center of the coil?



(Máté Vigh)

17. Let us have two straight parallel wires of length L and distance d, connected at both ends to form a rectangular circuit of size Ld (where $L \gg d$). In the middle of one of the long wires, there is an ideal resistor of constant resistance R; and in the middle of the other, an ideal voltage source of voltage U (independently of the load, and having zero internal resistance), along with a switch.

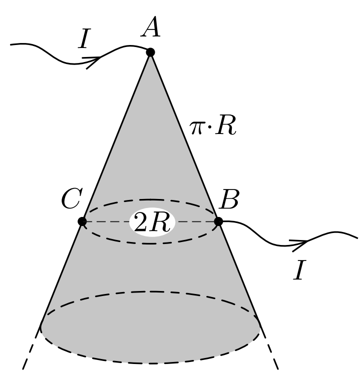
    How does the voltage on the resistor change in time (or the brightness, if the resistor is a bulb), starting from turning the switch on? How long does it take before the bulb turns on?

    What happens, if along with turning the source on (at the same moment, according to a comoving observer) we cut the wires on both short ends?

    How would things turn out if the wire would not be superconducting, but would have a given resistance per unit length?

(Máté Csanád)

18. A large conical shell is welded from a thin metal plate of thickness δ. (The cone is considered to be infinite compared to the other lengths in the problem.) Current I enters the cone at the vertex A, and leaves it at point B located on the generator of the cone. Find the magnitude and the direction of the current density vector at point C located oppositely to point B. It is known that the distance AB is $\pi \cdot R$, while the distance between points B and C (in space) is $2R$.



(Máté Vigh)

19. An electron of mass m and electric charge $-e$ moves around a fixed nucleus of charge $+Ze$ with relativisctic velocity. Derive the adequate version of Kepler's 3rd law, i.e. determine the relation between the orbiting time and the geometrical data of the orbit. Express the relation using a) time of reference frame, b) proper time of the orbiting electron.

(Orbiting time for non-closed orbits: the time between two successive perihelia.)

(Gyula Dávid and Dávid Szepessy)

20. Perturbation calculus in quantum mechanics can be improved by optimization. The starting point of the traditional perturbative method is a Hamiltonian $\hat{H} = \hat{H}_0 + \epsilon\hat{V}$, where the spectrum of the unperturbed $\hat{H}_0$ is known, and $\epsilon\hat{V}$ can be taken as a small perturbation. There is, however, some freedom in this decomposition, namely, part of the unperturbed operator with some weight factor can be separated off and joined with the perturbation. That way the total $\hat{H}$ does not, while both terms $\hat{H}_0$ and $\epsilon\hat{V}$ do change, and so the perturbative approximations for the energy levels get usually modified. Now, the latters can depend on the weight parameter, thereby we can try to minimize the error of the approximation.

As a simple example let us consider the harmonic oscillator, perturbed by a quartic potential, as

$$\hat{H}_0 = \frac{\hat{p}^2}{2m} + \frac{k(1+\lambda\epsilon)}{2}\hat{x}^2, \qquad \hat{V} = \epsilon\left(b\hat{x}^4 - \frac{k\lambda}{2}\hat{x}^2\right),$$

where m, k, b are constants, and ϵ, λ are dimensionless parameters. Now part of the quadratic potential, weighted by the factor $-\epsilon\lambda$, is included in the perturbation. Solve the problems below:

a) The case $\lambda = 0$ is a textbook exercise: calculate the n-th energy level to first order in ϵ!

b) Find, analytically or numerically, the optimal parameter λ whereby the perturbative expr-ession to first order best approximates the exact n-th energy level! Write down explicitly these expressions, or, compute them numerically for a few n's in a range of $\epsilon > 0$ up to a sufficiently large value! As a reminder, we do not expand by ϵ in $\hat{H}_0$, we only do so in the coefficient of $\epsilon\hat{V}$ to linear order.

c) Solve the time-independent Schrödinger equation numerically with a specific m, k, b setting for some n's, in the region for ϵ as studied in point b), for discrete values chosen sufficiently densely! Plot the E_n's as functions of ϵE_0, where $E_0 = \hbar\omega_0(n+1/2)$, $\omega_0 = \sqrt{k/m}$, together with the results from items a) and b)! Up to where is the optimized result a good approximation?

d) What can you say about $\epsilon < 0$? (Optional, the maximal point grade can be achieved without answering that.)

(Zoltán György and Géza Györgyi)

21. Calculate the values of the normalization constants K_Φ and K_Ψ, the expectation values and the standard deviations of the quantum number operator $\hat{N} = \hat{a}^+\hat{a}$ and those of the position operator $\hat{x}$ in the quantum states $|\Phi\rangle = K_\Phi \cos\left(\sqrt{w}\,\hat{a}^+\right)|0\rangle$ and $|\Psi\rangle = K_\Psi \sin\left(\sqrt{w}\,\hat{a}^+\right)|0\rangle$ of the one dimensional harmonic oscillator where $\hat{a}^+$ is the raising operator, $|0\rangle$ is the ground state of the oscillator, and w is a positive real number. Plot these quantities as functions of the parameter w.

(Gyula Dávid)

22. Convex combination of density operators is the following term

$$\hat{\rho} = \sum_{n=0}^{\infty} p_n \hat{\rho}_n, \qquad \text{where} \quad p_n \geq 0 \quad \forall n \quad \text{and} \qquad \sum_{n=0}^{\infty} p_n = 1,$$

and, of course

$$\mathrm{Tr}\hat{\rho} = 1.$$

Consider the following general Gaussian physical (positive semidefinite) density operator in coordinate representation

$$\begin{aligned} \rho_n(x,y) &= \exp\{-A_n\,(x-y)^2 - iB_n\,(x-y)\,(x+y) - C_n\,(x+y)^2 \\ &- iD_n(x-y) - E_n(x+y) - N_n\}, \end{aligned}$$

where the normalization factor is

$$e^{-N_n} = \sqrt{\frac{4C_n}{\pi}} e^{-\frac{E_n^2}{4C_n}},$$

as well as $A_n \geq C_n > 0$ and B_n, D_n, E_n are arbitrary real numbers.

Can we construct any physical density operator in infinite dimensional Hilbert space with convex combinations of Gaussian density operators? If so, how? If not, why not?

(Gábor Homa)

23. The von Neumann relative entropy of the state $\hat{\rho}_1$ with respect to the state $\hat{\rho}_2$ is defined as

$$S(\hat{\rho}_1 \mid \hat{\rho}_2) = \mathrm{Tr}\,\hat{\rho}_1(\log\hat{\rho}_1 - \log\hat{\rho}_2),$$

where $\hat{\rho}_1, \hat{\rho}_2 \geq 0$ and $\mathrm{Tr}\,\hat{\rho}_1 = \mathrm{Tr}\,\hat{\rho}_2 = 1$. Consider a Gaussian density matrix in coordinate representation of the form

$$\rho(x,y) = \exp\left\{-(A(x-y)^2 + iB(x+y)(x-y) + C(x+y)^2 + N)\right\},$$

where $A \geq C > 0$, $B \in \mathbb{R}$ and the normalization factor N guarantees $\mathrm{Tr}\,\hat{\rho} = 1$.

Calculate the relative entropy of a general Gaussian state $\hat{\rho}$ of the upper form with respect to the canonical state.

For fixed $\beta > 0$ the density operator of the canonical state (or Gibbs equilibrium state) is $\hat{\rho}_\beta = \exp(-\beta\hat{H})/\exp(-\beta F_\beta)$, where $F_\beta = -\beta^{-1}\,\mathrm{Tr}\exp(-\beta\hat{H})$ is the canonical free energy which ensures the normalization $\mathrm{Tr}\,\hat{\rho}_\beta = 1$. Then $S(\hat{\rho} \mid \hat{\rho}_\beta) = -S(\hat{\rho}) + \beta[\mathrm{Tr}(\hat{\rho}\hat{H}) - F_\beta]$. By defining the (Helmholtz) free energy of $\hat{\rho}$ as $F(\hat{\rho}) = \mathrm{Tr}(\hat{\rho}\hat{H}) - \beta^{-1}S(\hat{\rho})$ the relative entropy can be written as

$$S(\hat{\rho} \mid \hat{\rho}_\beta) = \beta(F(\hat{\rho}) - F_\beta),$$

where $S(\hat{\rho}) = -\,\mathrm{Tr}\,\hat{\rho}\log\hat{\rho}$.

Now let $T = 1/(k_B\beta)$, and $\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2$. Show that in this case

$$\rho_T(x,y) = \sqrt{\frac{m\omega}{2\pi\hbar\sinh(\hbar\omega/k_BT)}}\exp\left\{-\frac{m\omega}{2\hbar\sinh(\hbar\omega/k_BT)}\left((x^2+y^2)\cosh\frac{\hbar\omega}{k_BT} - 2xy\right)\right\}.$$

Determine the quantities $S(\hat{\rho} \mid \hat{\rho}_\beta)$, $F(\hat{\rho})$, F_β as functions of the above parameters.

(Gábor Homa)

24. a) Consider a prolate spheroid shaped cavity. The inner surface of the spheroid is an ideal mirror i.e. it reflects the electromagnetic radiation of any frequency without loss and distortion.

Fix two black holes in the focal points of the spheroid. There is no matter of any kind in the cavity except the black holes and their electromagnetic radiation.

The delay time the radiation needs to reach the wall of the cavity and go back to the sources can be neglected.

Determine the mass and temperature of the black holes as a function of time. Find an analytical solution.

What is the final state of the process?

Estimate the characteristic time of the process if the initial sum of the masses of the two black holes is $20M_\odot$ where $M_\odot$ is the mass of our Sun and the initial mass difference between the two black holes equals the mass of the Earth.

b) The geometry of the problem is the same as in the chapter a) but there is a homogeneous thermal electromagnetic radiation in the cavity. The temperature of the radiation is α) much higher, β) much lower than the Hawking temperature of the black holes. Determine and plot the temperature of the three subsystems as functions of time.

c) Now we have only one black hole fixed in the centre of a spherical cavity with ideal mirror on the inner surface. There is no matter of any kind in the cavity except the black hole and its electromagnetic radiation. The mirror surface is created at the moment $t = 0$.

But now we do not neglect the delay time the radiation needs to reach the wall of the cavity and go back to the center. Determine and plot the temperature and mass of the black hole as function of time.

(István Héjász)

25. In the special relativity theory the equation of motion of a mass point moving in an external scalar field $\Phi(x)$ is the following:

$$\frac{d}{d\tau}\left[\left(m + \frac{g}{c^2}\,\Phi(x)\right) u_k(\tau)\right] = g\,\partial_k\,\Phi(x),$$

where u_k is the four-velocity of the particle, c is the velocity of light, g is the coupling constant between the particle and the scalar field, and m is the rest mass of the particle (measured without the effect of the scalar field). The scalar field $\Phi(x)$ depends on the four-point x of the space-time.

This equation has many interesting solutions describing orbits of different shapes. Consider these worldlines to be geodesics of a corresponding curved space-time with adequate metric tensor, using the general theory of relativity. What is the metric tensor of such space-time?

Consider a special case when the scalar field $\Phi(x)$ is static, i.e. there exists an inertial reference frame in which the value of the field does not depend on the timelike 0th coordinate. Calculate the curvature tensor of the corresponding general relativistic curved space-time and the energy-momentum tensor of the hypothetic matter field causing the curvature.

Our additional assumption is that this matter causing the curvature is an unknown isotropic 'gas', i.e. its energy-momentum tensor is pure diagonal. What is the function $\Phi(\mathbf{r})$ in this case? What is the equation of the state of this 'gas'?

Add an extra assumption: let the scalar field $\Phi(r)$ central, i.e. its value depends only on the distance r between the particle and the origin. What is the energy-momentum tensor and the equation of the state of the isotropic matter causing the curvature in this case?

(Dávid Szepessy and Gyula Dávid)

26. Cellular automata are ultra-discrete classical models, where both the space and the time coordinate, but also the configurational space is discrete. Let us consider a one dimensional 'block cellular automaton': we have variables ψ_j, $j = 1, \dots, 2L$ arranged in a circle (periodic boundary conditions), such that each variable takes values from the set $X = \{0, 1, 2, \dots, N-1\}$. We have a discrete time variable $t \in \mathbb{Z}$ and the state of the system is $\Psi(t) = (\psi_1(t), \psi_2(t), \dots, \psi_{2L}(t))$. Time evolution is such that at each time step we perform simultaneous updates on two-site blocks, alternating the decomposition of the system into blocks. The local update rule is given by a map $U : X^2 \to X^2$, and we have

$$\Psi(t+1) = \begin{cases} \mathcal{V}_1 \Psi(t) & \text{for } t = 2k+1 \\ \mathcal{V}_2 \Psi(t) & \text{for } t = 2k, \end{cases}$$

where the alternating update operations are

$$\mathcal{V}_1 = U_{12}\, U_{34} \dots U_{2L-1,\,2L}, \qquad \mathcal{V}_2 = U_{23}\, U_{45} \dots U_{2L,\,1},$$

and it is understood that $U_{j,k}$ is the classical update step performed on 'sites' j and k.

- In a very simple case $N = 2$ and the map U is the permutation map: $U(x, y) = (y, x)$. Describe the dynamics in the system. How do you compute $\Psi(t)$ given the initial data $\Psi(0)$?
- Let us consider a generic U. What is the maximal speed of information propagation in this system? When is the dynamics reversible, in other words: when can we reconstruct $\Psi(0)$ given some $\Psi(t)$ with $t > 0$?
- Let us now choose some number $N \geq 3$ and the linear maps

$$U(x, y) = (x + y, x - y) \bmod N,$$

  where *mod* stands for 'modulo,' i.e. the remainder of division by N. For which N is this map reversible?
- The latest linear rule gives an ultra-discrete model of 'wave propagation'. Solve this model, i.e. express $\Psi(t)$ using $\Psi(0)$ in this system! The most elegant solution would require the use of algebra over finite rings or fields, but feel free to use the complex numbers as intermediate objects. What is the 'Green's function' of this model?
- The recurrence time T is defined to be the smallest non-zero time such that $\Psi(2T) = \Psi(0)$ for every initial condition. Choose $N = 3$ and the linear map above. Perform 'numerical experiments' to determine T as a function of the 'volume' L. When is T small, when is it large?

(Balázs Pozsgay)

27. The energy of the one dimensional, ferromagnetic Ising model is

$$E(S_1, S_2, \dots S_N) = -J \sum_{i=1}^{N-1} S_i S_{i+1},$$

where $J > 0$, $S_i \in \{\pm 1\}$, and $N \gg 1$. In the mean-field approximation the probability distribution is a product of one particle distributions, but it incorrectly predicts a phase transition. Let

$$P(\underline{S}) = \frac{P_{12}(S_1, S_2) P_{23}(S_2, S_3) \dots P_{N-1,N}(S_{N-1}, S_N)}{P_2(S_2) P_3(S_3) \dots P_{N-1}(S_{N-1})}$$

be the new variational trial distribution, where $P_i(S_i) = \sum_{S_{i+1}} P_{i,i+1}(S_i, S_{i+1}) = \sum_{S_{i-1}} P_{i-1,i}(S_{i-1} S_i)$

a) What is the variational free energy? b) Is there a phase transition? c) What is the correlation function?

(Máté Veszeli)

28. Uncle Joe lives close to the Large Supersonic Ambulance Car Collider. At least, this is how he calls the strange building complex in the middle of a large plane field. In fact a weird billionaire decided to supply the large cities with ambulance cars which run faster than the speed of sound, in order to speed up the transport of patients to and between hospitals. Well, so far what is achieved is a circular test track of many miles in diameter, concealed by a high, opaque fence. No light passes through the fence—but sound does all the more so. The ambulances run on the circular track at constant supersonic velocity, with sirens on, naturally. Fortunately it does not bother anyone.

Well, except for Uncle Joe. He lives on a farm a few miles from the test track and tells his friends about all his experiences at the pub on Monday night. His friends, of course, have heard a lot of strange fake news and passed on even more ones related to the real purpose of the test track.

'I am telling you,' he commences after the third mug of beer, 'this Muskle or whoever experimenting with teleportation behind the wall. You know in Captain Kirk way. Somehow beaming around these ambulances ...'

Well the others are not convinced. They learned from Carl Sagan that strong claims require strong evidence. So Uncle Joe is starting to prove it.

'You know I have excellent ears, perfect hearing. Already at the Isonzo, I could tell in advance from the grenade whistle where it was going to hit. So I sat in front of my house, closed my eyes, and started listening. From the direction and change of the sounds, I soon found that three ambulances, with constant siren, were chasing the track, two going in one direction and the third in the opposite. But alas! The voices of the two ambulances moving towards each other fluttered into a high pitch scream and then collided with a loud bang. That's it for these guys, I thought. But not! At the same moment, the two cars appeared at one closer point on the track, accompanied by another loud click, and as if nothing had happened, they began to move away from each other. Teleportation—I told myself—nothing else can this be!'

'Possibly quantum teleportation' remarked a cunning student.

'Like in the film "Back to the Future!". There, Doc's car had to be accelerated sufficiently for time travel, and maybe this ambulance teleportation could only take place if the cars are collided at high speed,' Uncle Joe meditated. 'I cant help saying it's a pretty inconvenient way to travel.'

'And what did the third ambulance do in the meantime?' the bartender asked.

'It just rolled on along the track, moving away from me. Well, somehow they got away with it now, I thought to myself. But then one of the cars that survived the previous collision and teleportation began to approach the previously unscathed third one, they collided and—like a wonder again—they were also teleported to a closer point on the track. And imagine, this is how it went all day long, repeating it on a regular basis! Collisions always took place at the same place, just as cars materialized at the same point after teleportation. If I could look behind the board, I am sure I could see the teleportation transmitter and receiver at these places ...'

'None of that is true!' the cunning student interrupted. 'I also scouted around there this afternoon to send up my drone to look behind the fence. And I saw that the test track is just a simple circle, there is no teleport, transmitter or receiver equipment at all. And most importantly, I also saw that there was nothing like the three fairy-tale cars going through dramatic adventures: a single ambulance makes boring laps on the track.'

Uncle Joe fell silent ashamed. The pub's crowd looked helpless. Who should they believe? Uncle Joe is an old, reliable member of the pub audience, he has never been caught lying. The cunning student, on the other hand, immediately showed the drone recordings. Even now, they would have wondered if the pub hadn't closed. For us, however, the only question remains:

How many times the speed of a supersonic ambulance circulating on a test track is the speed of sound in air? *Calculate numeric value of Mach's number.*

Additional question: What part of the circuit was 'jumped through' by Uncle Joe's ambulance during the teleportation?

Extra Additional question: Where would we arrive if we walked straight on the line connecting the departure and arrival points of the teleportation?

One more Extra Additional question: Uncle Joe told the same story on Tuesday night, but by then he firmly stated there were five ambulances circulating on the track, two of which repeatedly collided and teleported. And the cunning student once again presented a drone footage of the single supersonic ambulance he saw. Our questions, additionals and extras, are unchanged.

As a matter of fact on Wednesday evening Uncle Joe came up with a totally new hypothesis:

'I have again listened carefully today, and now I'm saying there was no teleportation behind the fence. This time, ambulance pairs showed up at a point near the track close to me and move apart, and at a remote point on the track, they collide and disappear without a trace. However, this does not happen at the same time as their appearance. I rather think a white hole and a black hole have been installed at two points in the circle, one spitting out the pairs of cars regularly, the other swallowing them at the same rate. Of course, it is also possible that a pair of a car and an anti-car will form and annihilate again and again.'

Naturally, the cunning student was again surely insisting on the boring theory about the single orbiting ambulance.

Just another Extra Additional question is quite simply: what happened at the track between Tuesday and Wednesday morning?

(Gyula Dávid)

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