THE 39th—11th INTERNATIONAL—
RUDOLF ORTVAY
PROBLEM SOLVING CONTEST IN PHYSICS
2008

The Physics Students' Association of Eötvös University, Budapest, the Roland Eötvös Physical Society and the Hungarian Association of Physics Students proudly announce the **39th** – and for the eleventh time **international** Rudolf Ortvay Problem Solving Contest in Physics, between 22 October 2008 and 3 November 2008.

Every university student from any country can participate in the Ortvay Contest. PhD students compete in a separate category. The contest is for individuals: solutions sent by groups of students are not accepted. The name, the university, the major, and the university year should be indicated on the solutions. Pseudonyms and passwords cannot be used: each contestant has to use his/her own name.

The problems can be **downloaded** from the webpages of the Ortvay Contest

http://ortvay.elte.hu/

in Hungarian and English languages, in html, LaTeX and Postscript formats, from **12 o'clock (Central European Time, 10:00 GMT), Wendesday, 22 October 2008**. The problems will also be distributed by local organizers at many universities outside of Hungary.

Despite all the efforts of the organizers, it may happen that some unclear points or misprints stay in the text. Therefore it is very useful to visit the webpage of the contest from time to time, as the corrections and/or modifications will appear there.

Each contestant can send solutions for up to 10 problems. For the solution of each problem 100 points can be given.

Any kind of reference material may be consulted; textbooks and articles of journals can be cited.

Each problem should be presented on (a) separate A4, or letter-sized sheet(s). The contestants are kindly asked to use only one side of each sheet. Solutions written by pencil or written on thin copy-paper will not be accepted as these cannot be faxed to the referees.

Computer programs appended to the solutions should be accompanied by detailed descriptions (what computer language it has been written in, how to use the program, which parameters can be set, what notations are used, how to interpret the output figures and graphs, etc.) They can be enclosed on floppy disks, or sent via email to the addresses below.

Solutions can be sent by mail, fax, or email (in LaTeX, TeX, pdf or Postscript formats). Contestants are asked not to use very special LaTeX style files unless included in the sent file(s). Electronically submitted solutions should be accompanied—in a separate e-mail—by the contents and, if necessary, a description explaining how to open it.

Postal Address: Fizikus Diákkör, Dávid Gyula, ELTE TTK Atomfizika Tanszék,
H-1117 Budapest, Pázmány Péter sétány 1/A, HUNGARY
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Deadline for sending the solutions: 12 o'clock CET (11:00 GMT), 3 November 2008.

Contestants are asked to fill in the form available on our webpage after posting their solutions. It will be used for identification of contestants and their solutions. **Without filling in the form, the organizers cannot accept the solutions! The form is available only on 2 to 4 November.**

The contest will be evaluated separately for each university year, according to the total number of points. The referees reserve the right to withhold, to multiply or to share some prizes. Beyond the money prizes given for the first, second, and third places, honorable mentions and special prizes for the outstanding solutions of individual problems can be awarded. This is why it is worthwhile sending even one or two solutions.

The announcement of the results will take place on 6 December, 2008. The detailed results will be available on the webpage of the contest thereafter. Certificates and prizes will be sent by mail.

Wishing a successful contest to all our participants,

the Organizing Committee: Gyula Dávid and József Cserti
The original Hungarian text is translated by Dezső Varga

1. **(Top Secret! Intercepted radio communication. CIA/1969/07/20/42/137)**

– Look there, just behind the retro rockets, what is that?
– I can't believe that! Some black domino, obelisk, or what... Just like in the 2001 Space Odyssey, except for the big grinning smiley painted on it... Let us stop right now!
– It may well be that you are the Commander, but you still not good at celestial mechanics. We can not stop here just like that. Let me tell you, that we are on our way towards the Mare Tranquillitatis, and if we break now down even the slightest, we never make it there – we might even crash down. And in space, everything moves, so one can not stop anyway...
– So why does that Thing stand there like that, and how?
– It is not standing. I just completed the measurements. It orbits around the Moon, just at the altitude of 137 kilometers.
– So let us get on the same orbit as that one, next to him!
– We do not have enough fuel to do that. And then we should be giving up the landing on the Moon. Let us do that on the way back! There is a bit of fuel left in the Lunar Module ascent stage, so that we can stand next to the Thing when we take off.
– All right, so we will find it with the radar on the way back.
– I have bad news for you. This Thing is not visible by the radar. I may say, it is a stealth object orbiting the Moon. We just realized it a moment ago because this black rectangle covered out a small part from the illuminated surface of the Moon. Seeing from downwards, we would not have found it in front of the background of the black sky.
– Wow. So how can we find it on the way back? You know what? Let us deposit a radio-buoy next to it!
– As yet, spaceships are not equipped with such stuffs.
– Too bad, in all Sci-Fi movies, this is the way to solve such situations. Do we not have something to substitute it?
– Well actually I have my little kid's rod-shaped electric torch from the boy scouts camp. I even have batteries—super batteries to last for a full week. It is said to produce ten Watts of power... if we deposit the lamp carefully outside, and push it with the appropriate speed, maybe we can put it on the same orbit as of that thing. Try to put it out such that it will continuously illuminate the black object. Then we will find the lamp with the radar—and find this UFO with the light of your lamp. All right, lean out, try to target that bright star... no not that one, the one next to it... and throw as large as you would do on a baseball match... Excellent!
– So did we manage?
– Closely. You took the right direction and orbit, and the lamp is about a hundred yards from the Thing. Unfortunately it started rotating, and it illuminates the rectangle only in every third second. Well anyway, it might even be more apparent when we search for it.
– It is so weird as it rotates...
– Well if you make it rotate not around one of its principal axis, but around an axis at 45 deg relative to the principal one, that is what you can expect. Quite lucky that the rotating axis points exactly towards the thing. OK make it back to the control board and let us continue the landing. We will be back in two days from now, and we take a closer look at what this Thing is. So, let's make it up to the Moon! Or rather, down to the Moon!

(Recording terminated.)

Comments from the analysing officer: it is a fact that the navigator's little son got his lamp back, and was showing it around for years at school. It is also a fact that the Lunar Module was hanging around at some place for a suspiciously long time, and made weird manouvres at the altitude of 137 km. But about the black Thing, no mentioning occured, neither in discussions, nor in official reports.

Evaluation: provided that the discussion above was not intended to confuse of interceptors, and records real events, then the only thing that one can think of is that the deposited torchlight was pushed off from proper orbit by an unknown physical effect, and even though the astronauts in the ascending Lunar Module indeed found the torch, it was not nearby the black stuff by then.

Suggested actions: proposed launch of top security research for revealing the unknown physical phenomenon. We have to figure out, which effect was pushing the torch and how far from its original position. We suggest (similarly to the idea proposed by our agent I. Asimov in the "Golden Egg" project) that we leak out the present document in a form of an international physics competition problem, so that maybe the unified intelligence of physics students would help solving the mistery. Besides, certainly we have to continue the direct search for the mysterious object, under the operation code name of "Needle in a Haystack".

Operation plan approved, leaking out postponed by 39 years.

Classified as Top Secret until June 20th, 2106.

(translated by Dezső Varga)

(József Cserti, Gyula Dávid)

2. The International Space Station crosses the night sky sometimes approaching the brightness of the Venus. Such passes are forecasted at the Heavens-Above homepage (`http://www.heavens-above.com`). Hint: choose "select from map or from database or edit manually" entry, where the observation point can be input with a precision of 100 m; then click on 'ISS' link). Watching the time distribution of such bright passes ('Prev' and 'Next' links backwards of forward in time) it can be seen that in a given time, the number of passes varies strongly. What is the reason for this variation? How is it precisely varying, if we neglect the atmospheric drag, and also neglect the ISS boosts (accelerations) which act against the drag?

   (translated by Dezső Varga)

   (Zoltán Kaufmann)

3. Jim Mortar, being a lazy kind of physics student, aims at crossing a long, straight, 10 meter wide road, on foot. His destination is on the other side, by that side of the road, but very far from the starting point. Jim is indeed lazy: he feels that if he follows the Highway Code, and crosses the road perpendicularly, he would pass a longer track than the shortest possible; this would mean an unnecessary extra distance. He wishes to reach the destination at a shorter path. At the same time, he is also careful: he does not want any car to hit him accidentally. From any point of the road, he can only see such section of the road, which a car makes in 10 seconds. He can move on food at a speed of $1m/s$. So that if he looks around at the start of the "crossing", and he does not see any cars approaching, he would be sure to safely arrive at the other side in 10 seconds if he moves perpendicularly. No cars appear – so he moves on. Soon he realizes that now there is no need to continue in the perpendicular direction, since he can turn slightly towards his destination point, and in the critical 10 seconds he would still be able to make it through the other side (if he continues straight in that direction). No cars still – so that he keeps more and more turning towards his destination, always only to the point that if he would see a car coming at any moment, he would go straight—keeping his momentary direction—and just make it to the other side in 10 seconds. As it turns out, there is no traffic, so no cars coming during the whole adventure. Therefore Jim follows a well defined curve, and finally—after a long time—he reaches his destination. How many seconds did he spare, compared to the situation as if he would have crossed the road perpendicularly?

   (translated by Dezső Varga)

   (Gábor Veres)

4. A circus equilibrist tries to climb on a long vertical rod. The rod is of length ℓ, with mass of m. At the beginning of the performance, the rod is lowered down using an elastic rope (of negligible weight) attached to the top of the circus dome. When the bottom of the rod touches the ground, the rope is of the length of 2ℓ. Without force applied, the unstreched length of the rope is ℓ, and obeys Hooke's law precisely.

   a) How high can the artist, of also mass m, climb on the rod, without the vertical equilibrium state of the rod becoming unstable? (For simplicity, assume that the size of the artist is negligible compared to ℓ.)

   b) At the height of half the rod length, the artist tips out, and starts sideways oscillation together with the rod. What is the period time T of the oscillations?

   c) The performance is repeated on a rotating stage. The rod is placed in the rotation axis, and the whole system including the rope rotates together with the stage. How high can the artist climb on the rod, if the perod time of the stage rotation is three times T (of the previous problem)?

   (translated by Dezső Varga)

   (Péter Balogh)

5. It is usually said that gyroscopic effect due to the conservation of the first wheel's angular momentum contributes decisively to the stability of a moving bicycle (anyone having held a rotating bicycle wheel in hand, may testify the magnitude of the appearing forces). Let us cancel this effect out in the following way: let us extend the axis of the first wheel, and on both side let us attach an extra wheel, which rotates on a ball bearing similar to the "normal" one. Assume that these "spare" wheels are slightly smaller in diameter so that they never touch the ground (but let them have the same moment of inertia).

   Now we can sit on the bike, start moving forward, and carefully spin up the "spare" wheels in a direction opposite to the normal one. If we are capable, the angular momentum of the "spare" wheels will cancel that of the normal one, therefore the total angular momentum is zero. This way there will be no gyroscopic torque. Let us try to steer the bicycle! Is it easier, or more difficult, than without the "spares"—or can we do that at all, without fallig off? Now let us try to spin the "spare" wheels faster in the "wrong" direction! This way the angular momentum and the gyroscopic torque are in the opposite direction relative to the usual (normal wheel) case. Can we steer the bike in this case? Explain the observations!

   (Provided that we lack either the time or special qualifications to construct the above discussed "spare-wheeled" bicycle, try solving the problem by pure theoretical considerations!)

   (translated by Dezső Varga)

   (István Csabai, Gyula Dávid)

6. Consider a particle of unit mass moving in the potential $V(x) = \frac{1}{2}x^2 + \lambda x^n$ (let n be integer for simplicity), and study its oscillations! The time of a full period as a function of amplitude A and parameter λ can be written as $T = 2\pi f(A, \lambda)$ where $f(A, 0) = 1$, and $\lim_{A \to 0} f(A, \lambda) = 1$.

   a) Prove that f depends only on a suitable combination of A and λ! Which is the first non-vanishing correction term in the Taylor-expansion of f? Discuss the odd and even n cases separately!

   b) In case of even n, write down the integral which determines the first correction, and then compute the cases $n = 4$, $n = 6$.

   c) Prove that the for odd n, the problem can be traced back to the case of even n, that is, prove, that with suitable choice of a and (even) n', the

   $$V'(x) = \frac{1}{2}x^2 + a\lambda^2 x^{n'}$$

   potential gives the same correction as the original $V(x)$. (*Hint:* Landau: Theoretical Physics I. Mechanics 12.)

   (translated by Dezső Varga)

   (Balázs Pozsgay)

7. Three bugs with negligible masses ($m = 0$) walk along a circle of radius r. The frictional forces acting on the bugs are proportional to their velocities, the coefficient is denoted by γ. The bugs drive themselves with a constant force in time, thus when they are unperturbed, their velocities are constant. These velocities form the consecutive terms of an arithmetic series. However, we perturb the bugs: we connect them pairwise along the chords of the circle with springs that obey Hooke's law. The unstreched lengths of the springs are zero, they have no mass, and their spring constant is D. Investigate the motion of the bugs after a sufficiently long time (the motion is henceforward fixed to the circle)! (Attention: theoretical problem, do not experiment with animals!)

   (translated by the Author)

   (Máté Maródi)

8. What is the relative increase of the deformation of a prism, of area A, height l and Young modulus E, when we not only place a body of mass m and of area A on it, but we let this body fall on the prism from a height of h? What can we say if we take into account not only the Young modulus E of the prism, but also the Poisson-number σ?

   (translated by Dezső Varga)

   (József Cserti)

9. Two cylinders of same size, of radius R and length L are welded together, such that we get a cylinder of length $2L$ and radius R. One of the cylinders is made of iron, the other of copper. Which are the frequencies of the elastic standing waves in the system?

   (translated by Dezső Varga)

   (József Cserti)

10. Give an estimate of the impact of certain anthropic effects (mining, building construction, popularity increase, filling up water reservoirs, or others not considered by the author of this problem) on the change of the length of one day on Earth, since the birth of Sir Isaac Newton! Give the values in seconds, separately and in total.

   (translated by Dezső Varga)

   (György Hetényi)

11. A metal layer of thickness D, bounded by planes, is embedded in an elastic bulk material. Elastic waves are falling on the layer from a direction perpendicular to its plane. What is the condition of the case when the transmisson of the layer for the longitudinal and transversal components of the wave is the same?

   (translated by Dezső Varga)

   (József Cserti, Gyula Dávid)

12. A spherically symmetric source emits sound at power P, of very low frequency (that is, infra-sound at very large wavelength $\lambda = c_{sound}/f$.

   a. What is the amplitude of the pressure variation at a distance of R from the sound source?

   b. Let us hang an object of box (cuboid) shape relatively far from the sound source, such that it may move freely in any directions (one side of the box is perpendicular to the direction of the sound source). Estimate what is the amplitude of the vibration caused by the sound falling on it! (Consider only stationary, constant amplitude oscillations.)

   (translated by Dezső Varga)

   (Gyula Szokoly)

13. Designing audience rooms or home cinema studios it is often a problem, that depending on the shape of the room some frequencies of the soundwave spectrum are strengthening, which leads to the deterioration of the sound quality. The problem can be solved by placing some inverted loudspeakers (sound traps) at some points in the room. Design an equipment which is able to damp a given sound with frequency f only by using a tube and a rigid box! Where should be these traps placed?

    (translated by the Author)

    (Gergely Fejős)

14. Let us prepare a "super sandwich" out of metal layers: we deposit subsequent layers of metal type A at thickness d_A, and then metal type B at thickness d_B (always assumed to be infinite planar layers). The longitudinal sound velocity in metal type A is c_A, and the sound velocity in metal B is c_B. The sandwich contains alltogether $2N+1$ layers of the sequence ($ABABA...ABABA$). Finally we attach to both ends an infinitely large bulk of metal type B. Perpendicular to its plane, a longitudinal sound wave is emitted on the sandwich. Calculate the transmission and reflection coefficients of the total sandwich as a function of sound wavelength!

    (translated by Dezső Varga)

    (József Cserti, Gyula Dávid)

15. We observe a mirage over the Hortobágy (region on the Hungarian Great Planes) slighty above the horizon. Let us assume that the refraction index n of air is a function only of the altitude z. What should be the conditions imposed on function $n(z)$ so that a mirage will appear? When will it be upright (same direction as the object), and when will it be an upside down (swapped) picture?

    (translated by Dezső Varga)

    (Zoltán Kaufmann)

16. Three mathematicians, Andrew, Anna and Attila are discussing the following problem: "*How much does one need to thicken a coin such that when randomly dropped on the table, it will stand on it's rim by a probability of 1/3?*" They try to simplify the issue to the most: considering a homogeneous regular cylinder, assuming that when it hits the ground, it stops moving. They quickly agree that this way the final state is simply calculable based on the information about the initial state. Then they start discussing the more tricky question: "*What is actually the random distribution of the initial state?*" They agree—on the basis of equal sharing of configurations—that it must be uniform, but disagree on in which coordinates the uniformity must be realized:

    Andrew insists that the only relevant coordinate is the angle between the cylinder axis and the vertical direction, and therefore suggests that the uniformity is to be considered in this coordinate distribution.

    Anna's opinion is that one must take into account the fact that the space has three dimensions; according to her the direction of the cylinder axis is to be defined on a unit sphere, and the uniformity of the distribution to be required on this sphere.

    Having heard his colleagues, Attila vaguely remarks, that some time ago on the physics course he heard something about Euler's angles, which give a unique description of any object, and suggests that the initial state distribution of the cylinder can eventually be uniform in these Euler's angles.

    Determine in all these three cases the probability that the coin will finally stand on its rim as a function of the height/radius ratio!

    And as a conclusion, make justice among the mathematicians!

    (translated by Dezső Varga)

    (Péter Kómár)

17. We are given n balls, which are apparently equal and not too heavy. All of them are of equal weight, except for one (we do not know if the one is heavier or lighter). Our aim is to separate out this different ball in the shortest possible time. There are two instruments at our disposal: one is a normal table scale with a display; the other is a double-arm scale (both are ideal, i.e. infinitely precise and can hold infinite weight).

    a) On the table scale, we can measure the balls one by one. What is the expected time of finding the different ball, if one measurement takes T time? What is this time in the limit of $n \to \infty$?

    b) Determine the minimal number of measurement (denoted by $N(n)$) by which using the double-arm scale we can surely find the different ball!

    c) If one measurement on the double-arm scale takes on average $C \cdot T$, at which value of C should one use the double-arm scale at fixed n?

    d. Let n be 40, and $C = 51/11$! Show that in this case the double-arm scale is favourable, and describe the measurement procedure in all details from beginning to end!

    (translated by Dezső Varga)

    (Gábor Homa)

18. Alice, Betty and Claire are triple twins, and by their feminin kink, keep competing with each other all the time: so it happens also in this case. Their games are always about chances. Their present "gamble" is the following: they have 10 glasses of water at their disposal, filled with water of same amount; also the glasses are all alike. Three glasses contain water at $T_1 = 5$ °C and seven glasses contain water at $T_2 = 35$ °C; it is indistinguishable which is at which temperature. Each girls choose one glass. They also have three technically identical refrigerators, so that the water freezes in each of them in time $t(T_1)$ and $t(T_2)$ respectively. The conditions of cooling are such that for each refrigerator $3 \cdot t(T_2) < 2 \cdot t(T_1)$. The girls place their glasses in the refrigerators all at the same time, and whose water freezes first, is the winner. To avoid tie game, two of the refrigerators are placed into an automatically controlled spaceship at relativistic speed, programmed differently. The spaceships of Alice and Betty start immediately after having the glasses placed inside, and after having the refrigerators switched on. They accelerate identically in the beginning: their rapidity seen from the coordinate system of Earth varies as $\beta(t) = k \cdot t^{1/4}$ as a function of time. Alice's spaceship accelerates for a time of $t(T_2)/4$, stops instantly, then accelerates backwards according to the same function as before, then arrives at Earth after a time of $t(T_2)/2$ measured from the moment of start. Here it stops again instantly. The other spaceship, Betty's one, accelerates similarly, but stops already after a time of $t(T_2)/8$, turns back and arrives after a total flight time of $t(T_2)/4$. The third refrigerator stays on Earth, but switched on at the same time as the spaceships leave.

After the arrival of the spaceships, the girls are excitedly awaiting for the water to freeze...

a) Determine all possible outcomes of the contest, and give the probability of each outcome!

b) What is the probability of such glass of water to be placed inside a given refrigerator which freezes earlier?

c) How much did Alice's and Betty's glass got older during the time of $t(T_2)$? Give the result as a function of k and $t(T_2)$. What conditions should one impose on k, so that the present problem makes sense?

d) Extra question: What does the problem have to do with Tansania?

(translated by Dezső Varga)

(Gábor Homa)

19. Recently popular way of file-sharing via the Internet is the use of torrents. One torrent allows many files (defined in the torrent) simultaneous (or interleaved) download, and for this reason most files are only partially (unusably) available.

It is reasonable to assume, that the downloading program which manages the torrent downloads the parts of files practically randomly. Consider a torrent that has many files of the same length. Based on the above assumption, give a theoretical prediction on what is the distribution of the length of the files as a functon of a given average 'global" readiness fraction (global fraction of total data to be downloaded).

Extend this theoretical relation, for the case when the torrent has many files with different length! Test the theory with comparing the prediction with actual measurements made on real torrent download, available at the following web address:

`http://pkomar.web.elte.hu/Ortvay2008.html`

(translated by Dezső Varga)

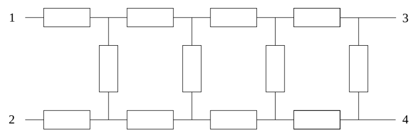
(Péter Kómár)

20. Direct current flowing in an electrically neutral conducting ring gives rise to a magnetic field. In a coordinate system rotating around the symmetry axis of the ring this leads to the appearence of an electric field which points towards the ring. This means that the ring has a nonzero electric charge in the rotating coordinate system. Explain the phenomenon and check charge conservation!

(translated by the Author)

(Gyula Bene)

21. Let us construct a resistor ladder of N steps from $3N$ piece of resistors, each of R resistance. Let us put this ladder on a Moebius-tape! The simplest way to realize this is (e.g. for the case of $N = 4$) that we contact the terminals 1 and 4, and contact terminals 2 and 3 on the Figure below.

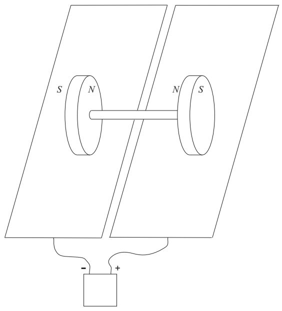


What is the resistance between any two points of the resistor circuit? Study the expressions corresponding to the lowest values of N (1 and 2) numerically!

(translated by Dezső Varga)

(Gábor Széchenyi)

22. Two strong, cylinder shaped ceramic magnets are attached to a thin conducting rod, and placed on a flat table. Both magnets has their North faces (N on the figure) towards the rod. The magnets are covered with an electrically conductive layer. On the table, two conducting strips are placed, insulated from each other, each touching one of the magnets.



a) If we apply direct current on the strips, the system of magnets and axis will start rolling. What is the direction of the movement, if the positive voltage is applied on the right side strip?

b) Discuss the behaviour of the system from the point of view of theory of momentum and angular momentum conservation.

(translated by Dezső Varga)

(András Juhász)

23. Let us consider a metal sphere put into a homogenous electrostatic field. Calculate purely the energy belonging to the sphere, ie. the sum of the shared charges' self energy and the interacting energy between them. Based on these results, show that if the charges on the surface are fixed somehow, then the interacting energy between a dipole put into the sphere and the shared charges is not in line with the laws of electrostatics! What has to be changed in order to preserve the consistency with electrostatics? What is the physical reason for that?

(translated by the Author)

(Gergely Fejős)

24. Let us construct the transfer-matrix formalism of magnetic focusing systems of high energy particle accelerators! Let us follow the particle's trajectories in the direction of the acceleration. In this direction the length travelled is practically the total path length (that is the angle between the real and the "wished" trajectories is very small). Let us use this path length as the parameter describing the trajectory.

Among the trajectories, there is one special: the one we wish the particle to go along. The particle will indeed follow this trajectory, provided that we let it start from the right place in the right direction. Let us call this the design trajectory.

The trajectory is uniquely defined, if we know its position and direction with respect to the design trajectory. Let us denote the particle's position in the plane perpendicular to the design trajectory by x and y (horizontal and vertical). The direction can be given as the derivative of x and y with respect to the path length parameter, denoted by x' and y' (according to the assumptions above, they are both much smaller than 1).

In case that in the beginning, $(x, y, x', y') = (0, 0, 0, 0)$, the particle will continue all the way at the these coordinates – by the definition of the design trajectory. Now if the above vector $a = (x, y, x', y')$ is not zero, then to first approximation at any given place further down the trajectory, the a_{new} will be given by a linear combination of elements of a: $a_{new} = Ta$, where let us call the matrix T the transfer matrix of the system between the new and original point. Clearly if we put two independent systems behind each other, the product of their transfer matrices will give the transfer martix of the combined system.

Let us determine the transfer matrices of the following systems:

a) Free propagation. The particle runs along a straight line without external force, down the vacuum-pipe.

b) Magnetic quadruple. Let us prepare on a short distance (how short: this is also to be determined in which approximation we consider the lenght "short") a quadruple-like magnetic field, with axis parallel to the design trajectory, and let us bring the particle through that. What will be the focal length of the system in the different directions (any angles in the (x, y) plane)?

c) Let us place a quadruple, a free propagation of given length, and again a quadruple behind each other. Can such a system focus in all directions? What is the focal length of this "magnetic lens"? What is the achromatic error?

d) What is the transfer-matrix of a homogeneous field of a given length? (Attention: if we let two particles run parallel in a homogeneous magnetic field, they will not run in parallel – that is, the transfer matrix is not trivial).

Useful remark: Considering the fact, that the trajectory of a relativistic particle depends only on the magnitude of the momentum, our calculations, if done in the non-relativistic limit, will automatically be correct in the relativistic case too...

(translated by the Author)

(Dezső Varga)

25. What should be the design diameter of a copper wire, initially at the temperature of 40 °C, so that under a current of $I = 12\,000$ A, applied for 0.5 seconds, it does not heat up more than temperature of 180 °C ? Consider that the resistance of the metal varies with temperature as $\varrho(T) = \varrho_0\,(1 + \alpha T)$, where ϱ_0 and α are known constants for copper.

Let us consider now the other "extreme" case, which is an infinitely thin, very long superconducting wire. In the wire a constant current is flowing. Let us assume that all thermal properties of the wire are independent of the wire temperature, and also that above the superconducting temperature threshold T_0 the resistance of the wire is constant (though these assumptions are far from reality). Let the initial wire temperature be absolute zero, and assume that the heat exchange with the outside environment is negligible. If suddenly on asmall section the superconductivity is lost, the temperature begins to rise. After a while, the whole wire will heat up: the wire "quenches", which poses one of the main difficulties at accelerators such as the LHC. How fast the quench-wave will spread over the wire?

Suggestion: try to find the solution $T(x,t)$ of the temperature conductivity equation in a "travelling wave" form, that is as $T(x - vt)$.

(translated by Dezső Varga)

(József Cserti, Dezső Varga)

26. The little Luke Skywalker does his usual homework at the deserted, dreary land of his uncle Owen: he installs solar cells over an area which the cells must compeletely cover. Young Luke knows very well how far the Two Suns of Tattooine are from his planet, and knows also the diameter of these stars; furthermore he can measure during his work with his instruments the solar energy falling on unit area during one minute of time. He starts wondering: would it be possible to determine the surface temperature of the Suns, assuming that both stars are radiating to good approximation as black bodies? Then keeps thinking, if he needs to know for this the radiation entropy for each stars for a given volume? Fortunately a benignant wookie hangs around there, and readily helps young Skywalker out. He lets him know the radiation entropy ratio of the two stars, on the elementary reversible process, over the volume of the Far-Far-Away Galaxy. Help the youngster Jedi, knowing these data, to determine the surface temperature of each Suns, and their radiation entropy calculated over the volume of the Galaxy! What was the ratio that the Wookie gave to him?

(translated by Dezső Varga)

(Gábor Homa)

27. Let us assume, that the LHC will produce a neutral stable black hole at rest with a mass of 2 TeV. What will happen? Let us treat the black hole as a classical object which is only involved in the gravitational interaction. What will be its trajectory? How does its mass change in time?

(Christian Hölbling)

28. The simplest description of filament polymerization in solution involves the sequential attachment of monomers (of size a and concentration c) to the end of a growing filament with a second order rate constant k_{on} and the detachment of these monomers from the filament end with a first order rate constant k_{off}. Filament growth can be slowed down by imposing a counter force, mediated by a small frictionless obstacle that is pushed against the end of the filament at a force F. Assuming that the filament is immobile (held at a fixed position or has a very large friction coefficient) and remains straight (cannot buckle) all the time, determine the force F_{stall} necessary to stop the growth, and also the dependence of the speed of growth on the counter force and the monomer concentration.

(translated by the Author)

(Imre Derényi)

29. A well established tool for investigating one-dimensional stationary quantum mechanics problems is the transfer matrix method. Consider a wave function

$$\psi(x,t) = \phi(x)\, e^{-\frac{i}{\hbar}Et}$$

propagating in a continuously changing potential $V(x)$ and approximate the potential with a step function consisiting of constant values on small intervals Δx. Using the transfer matrix method calculate the change in the components of the wave function propagating to the right and to the left on an interval Δx. Investigate the limit $\Delta x \to 0$ and deduce the system of differential equations describing the two components of the wave function. Which differential equation is satisfied by the components of wave function separately?

(traslated by Katalin Kulacsy)

(Gyula Dávid)

30. Alice and Bob are discussing the muon decay. They both agree that a muon decays into an electron, a muon-neutrino and an electron-antineutrino. They disagree on the angular distribution of the produced electron. Alice has heard somewhere (unfortunately, she does not remember where) that the angular distribution of the electron is not isotropic but it depends on the spin of the muon. She even remembers that the distribution depends on the cosine of the θ angle between the spin of the muon and the direction of the outgoing electron in the form $a + b\cos(\theta)$.

Bob claims that this is impossible. His argument is quite simple: Let's take a great number of muons with random spins. Approximeately half of the spins will point up, the other half down. Therefore, the distribution of half of the produced electrons contain a $\cos(\theta)$ term, the other half a $\cos(\pi - \theta)$ term. The total distribution will not be isotropic. This is impossible since there was no distinguished direction, the original setup has rotational symmetry. Help them resolve the controversy! Can the electron distribution have the form $a + b\cos(\theta)$ or did Alice remember incorrectly? How can we get an isotropic distribution for the decay of a muon gas consisting of lots of mouns?

(translated by the Author)

(Sándor Katz)

31. The low frequency excitations of Bose-condensed trapped gases result as the solution of the following eigenvalue-equation:

$$\omega^2 f = -\frac{1}{m}\nabla\left[(\mu - V(\mathbf{r}))\nabla f\right] \tag{1}$$

(assuming mass m and chemical potential μ as given). Let ρ, z and ϕ denote the usual cylindrial coordinates, and let $V(r)$ denote the axially (cylindrically) symmetric harmonic oscillator potential, as it is usually applied in experiments:

$$V(\mathbf{r}) = m\omega_\perp^2\rho^2/2 + m\omega_z^2 z^2/2.$$

Let us introduce the following parameters: $a = \sqrt{\frac{2\mu}{m\omega_\perp^2}}$, $b = \sqrt{\frac{2\mu}{m\omega_z^2}}$, and $c = \sqrt{\frac{\mu}{m}}$.

a) Prove that the solutions of equation (1) are functions f of the following form:

$$f = \rho^{|m|}e^{im\varphi}z^\alpha \prod_{i=1}^{n}\left(1 - \frac{\rho^2}{a^2+\Theta_i} - \frac{z^2}{b^2+\Theta_i}\right), \tag{2}$$

where n, m are integers, and $\alpha \in \{0, 1\}$.

b) How to express ω^2 with the quantum numbers m, n and α, and with the parameters a, b, c, Θ_i?

c) Which is the (ordinary, algebraic) system of equations, which Θ_i must fulfill, so that the form in (2) will be the solution of equation (1)?

(translated by Dezső Varga)

(András Csordás)

32. A granite sphere of radius R contains uranium in small quantity, in a homogeneous distribution. The decay chain of uranium contains the Radium (Ra), and its daughter element, the Radon (Rn). The half-life of Radon is 3.8 days. The half lifes of the daughter elements of Radon are very small compared to this timescale, and its ancestor elements decay on much longer timescales. The complete Radium and Radon activities of the sphere (A_{Ra} and A_{Rn}) are well measurable by detecting their gamma radiations. In a radioactive equilibrium, we would get $A_{Ra} = A_{Rn}$, but part of the gaseous Radon diffuses out of the sphere before decaying, therefore we get the ratio $\alpha = A_{Rn}/A_{Ra} < 1$. We determine the value of α with our gamma-detector. Knowing this value, calculate the unknown diffusion coefficient D of the Radon! What is the value of D, if α is 0.1, 0.5 or 0.9?

(translated by Dezső Varga)

(Gábor Veres)

33. In an idealized supernova explosion at a certain location there are N ^{101}Nb (niobium) nuclei and $M = 0$ ^{102}Nb nuclei and there is a neutron flux going through which is time dependent, exponentially decreasing during $2s$. The neutron flux decreases from the initial value down to the 1/3 of it during this $2s$. This neutron flux transforms the nuclei by neutron capture, so the ^{101}Nb will be transformed to ^{102}Nb. The capture cross section is so large that it transforms $N/11$ of ^{101}Nb nuclei per second at $t = 0$. From the ^{101}Nb there will be ^{102}Nb nuclei but these can be created in two states: ground state and excited isomer state, the latter has 30% probability. (All the other excited states are neglegted now.) The isomer state will decay by emitting a gamma photon, and its half live is $1.3\,s$. The half live of the ground state of ^{102}Nb is $4.3\,s$, and for the ^{101}Nb $7.1\,s$. How does the number of ^{102}Nb nuclei depend on time?

(translated by the Author)

(Ákos Horváth)

34. We aim at a "discovery" of a phenomenon; we ask the question what is the statistical significance of the phenomenon to exist, or in other words, what is the probability that we "see something" when nothing happens actually. Let us consider a specific method, when we count the occurence of the phenomenon: assume that we observe it M times. The same phenomenon may apparently occur for "other" reasons, let us call this background (mistaken measuremens or random coincidence of external conditions). Let N be the number of background events, and for now on let us assume that we know very precisely the expectation value of N based on external informations.

In this case then, we measure M events, with a background of N. N follows Poissonian distribution (which may be approximated with a Gaussian for simplicity). The average deviation (RMS) of N is $\sqrt{N}$. M is "large enough" if it is larger than N by a multiple of its error: $M > N + s_0\sqrt{N}$. Here s_0 refers to the "significance", i.e. we can determine the (small) probability $p(s_0)$ that M is at least $N + s_0\sqrt{N}$, provided that there is only background (and no extra phenomenon). As an example, $s_0 = 5$ corresponds to $p(5) \approx 5.5 \cdot 10^{-7}$, that is with a probability of $1 - p(s_0)$, we have "discovered" the occurence of the phenomenon. The value above is often quoted as "five sigma" significance, and it is the generally accepted level of discovery.

After this generally well known introduction, let us consider the following case: we do not have only a single kind of measurement result, but we measure as a function of a relevant parameter (mass of the particle to be discovered, frequency of extraterrestial radio waves, etc). As a function of the parameter, we subdivide the data in bins (histogram), and we determine the number of occurences M_i in each of these bins. We subdivided the data into K intervals, so i runs from 1 to K. In each bin there is a known average background. In this case, we do not know, where is the interesting phenomena: for this reason we try to find among the values any which is "jumping" out high. That is, for any i, we look for M_i which is higher enough than the background N_i.

Let us make two simplifications: let the expectation value of the background N_i be independent of i (and denote it by N). Also let us assume that if there is anything to "discover", it may be anywhere (at any i), but it will appear only in one single measured point. Let us denote then the largest measured M_i by M. We have "discovered" the thing if $M > N + s_1\sqrt{N}$.

a) What should be the value of s_1, if we require that with only background (without extra particle) the probability of fulfillment of the above inequality is $p(s_0)$? Write down the relation between s_1 and s_0 (which clearly depends on K).

b) An honest physicist has completed the measurements, but considers them not "precise enough": so that she/he applies honest methods to improve quality. She/he finds good reasons to select from the data, removing the "unprecise", or "erroneous" ones (where the background was smaller or the instruments were more precise in some part of the measurement), but does this (unconciously) on the basis if the outcome is better with these selections or not. If not, she/he does not take this specific part of the data out (this was not the good way of selecting, she/he thinks) and makes selection based on other conditions.

Let us model this situation in the following way: let us separate the measurement in to two random samples (every single measurement falls into one or the other bunch with 50% probability). This way the background N_i' has an expectation value of $N/2$ for both data sets. Now let us choose the one dataset in which the new largest number, M', is greater. This case we can define the significance s_2, that is, when $M' > N/2 + s_2\sqrt{N/2}$. What value should s_2 have, so that with background only, we get at least this large M' with probability of $p(s_0)$?

Remarks: The "good-willed data selection" mentioned in point b) is difficult to pinpoint in specific cases; sometimes it turns out only after years that the result was faulty. Good example is the "*Split A2 resonance*" case from 1967. In the results from Phys.Lett.25B(1967)44. paper, the phenomenon was found at 5 sigma significance, whereas later on it was cleared out that it was mere statistical fluctuation. There the selection was based on measurement accuracy (see Fig. 2. of the paper), and half of the data where the splitting was not present was simply got rid of. Further interesting issue is, that the phenomenon was confirmed by two other experiments soon after discovery... (see Phys.Lett.B31(1970)397.)

(translated by the Author)

(Dezső Varga)

35. In an Universe uniformly filled with background radiation of temperature T_0, there is a black hole of mass M_0. It absorbs continuously the photons of the cosmic background, at at the same time it emits Hawking's radiation (this is a thermal, i.e. Planck-spectrum distributed electromagnetic radiation, which is of a characteristic temperature inversely proportional to the mass of the black hole). Write down the differential equation which determines the time evolution of the black hole mass! Consider now also the fact, that the temperature of the background radiation decreases, in inverse proportion to the lifetime of the Universe! Solve the differential equation in the two special limiting cases, that is when the characteristic temperature of the black hole is a) much lower; b) much higher, than the temperature of the background radiaton!

When and how does the transition between the two limiting cases happen? Try to work with realistic parameters! (*Suggestion:* use the Planck units during the calculations!)

(translated by Dezső Varga)

(Gyula Dávid)

36. “The Dawning Star is setting down...” says the well known Hungarian folk song—reflecting the keen minds of people of ancient times, distinguishing the Venus at dawn from the Venus at the evening (i.e. the Evening Star). For a long time, it did not occur to anyone that the above statement is an astronomical nonsense: when the Venus appears at dawn, it never goes down, but always rises up (until it vanishes in the light of the rising Sun). When it moves downwards, then it is evening time, and it is called the Evening Star.

Few enthusiastic astronomers—after having seen many meteorites, having drunk large quantities and having sang songs all night—finally realized this issue. They simply assigned the contradiction to the observation incapabilities of the singers of ancient times.

Recently however, a publication appeared in a national ethnographological journal, arguing that the above cited folk song proves the excellent observation capabilities of the singers of ancient times, and at the same time proves the authors' theory that Mankind—but Hungarians at least—originate from a planet of Sirius. The text of the folk song is a strong astronomical evidence of thir thrilling theory.

Biological evidences (e.g. lack of thick hair on Homo Sapiens) suggests that the Ancient Earth had similar surface temperature as the present Earth; furthermore, the atmosphere composition and pressure could not be very much different. Other evidences quoted by the authors suggest that one day on Ancient Earth was 42 hours instead of 24. No further information is available at present. (Of course, 42 is The Answer, what else...)

The authors quote that such objects as the Phobos moon of Mars can not be considered as Dawning Star—i.e. no natural or artifical objects which on orbit moves faster around its parent planet than the revolution time of the planet itself, and which would rise on West and would set on East. Such an object would be also visible well before dawn while crossing the night sky. The “real” Dawning Star rises not much before dawn, and then—as the folk song says—reconsiders the issue and sets closely at the same place.

Unfortunately the authors of the publication did not allow the full description of the case. Let us do it for ourselves! Figure out the structure of the Ancient Home-Solar-System, at least its main features! Furthermore, calculate how high the original Dawning Star could rise, for how long it was visible before setting.

(translated by Dezső Varga)

(Gyula Dávid)

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