THE 33rd
– 5th INTERNATIONAL –
RUDOLF ORTVAY
PROBLEM SOLVING CONTEST IN PHYSICS
2002

The Physics Students' Association of Eötvös University, Budapest, the Roland Eötvös Physical Society and the Hungarian Association of Physics Students proudly announce the **33rd** – and for the fifth time **international** –

Rudolf Ortvay Problem Solving Contest in Physics,
from 31 October 2002, through 11 November 2002.

Every university student from any country can participate in the Ortvay Contest. PhD students compete in a separate category. The contest is for individuals: solutions sent by groups of students are not accepted. The name, the university, the major, and the university year should be indicated on the solutions. Pseudonyms and passwords cannot be used: each contestant has to use his/her own name. The problems can be **downloaded** from the webpages of the Ortvay Contest

http://ortvay.elte.hu/

http://www.saas.hu/ortvay

in Hungarian and English languages, in html, LaTeX and Postscript formats, from **12 o'clock (Central European Time, 11:00 GMT), Thursday, 31 October 2002**. The problems will also be distributed by local organizers at many universities outside of Hungary.
Despite all the efforts of the organizers, it may happen that some unclear points or misprintz stay in the text. Therefore it is very useful to visit the webpage of the contest from time to time, as the corrections and/or modifications will appear there.
Each contestant can send solutions for up to 10 problems. For the solution of each problem 100 points can be given. *Each problem should be presented on (a) separate A4, or letter-sized sheet(s). The contestants are kindly asked to use only one side of each sheet. Solutions written by pencil or written on thin copy-paper will not be accepted as these cannot be faxed to the referees.*
Any kind of reference material may be consulted; textbooks and articles of journals can be cited. Computer programs appended to the solutions should be accompanied by detailed descriptions (what computer language it has been written in, how to use the program, which parameters can be set, what notations are used, how to interpret the output figures and graphs, etc.) They can be enclosed on floppy disks, or sent via email to the addresses below. Solutions can be sent by mail, fax, or email (in LaTeX, TeX or Postscript formats—or, if they contain no formulae, in normal electronic mail). Contestants are asked not to use very special LaTeX style files unless included in the sent file(s).

Postal Address:

Fizikus Diákkör, Dávid Gyula,
ELTE TTK Atomfizika Tanszék,
H-1117 Budapest, Pázmány Péter sétány 1/A
Fax: Dávid Gyula, 36-1-3722775 *or* Cserti József, 36-1-3722866
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Deadline for sending the solutions: 12 o'clock CET (11:00 GMT), Monday, 11 November 2002.

Contestants are asked to fill in the form available on our webpage after posting their solutions. It will be used for identification of contestants and their solutions. **Without filling in the form, the organizers cannot accept the solutions!** The contest will be evaluated separately for each university year, according to the total number of points. The referees reserve the right to withhold, to multiply or to share some prizes. Beyond the money prizes given for the first, second, and third places, honorable mentions and special prizes for the outstanding solutions of individual problems can be awarded. This is why it is worthwhile sending even one or two solutions.
The announcement of the results will take place on 5 December, 2002. The detailed results will be available on the webpage of the contest thereafter. Certificates and money prizes will be sent by mail. We plan to publish the assigned problems and their solutions in English language—to which the contribution of the most successful participants is kindly asked. The volume is planned to be distributed all over the world with the help of the International Association of Physics Students, as well as the contestants themselves. We hope this will help in making the contest even more international. Wishing a successful contest to all our participants,

the Organizing Committee:

Gyula Dávid, Attila Piróth, József Cserti
(Eötvös University, Budapest, Hungary)

1. Build a Hollywood-style bomb, which explodes upon cutting through any of its wires. (Model: a bulb is lit. Otherwise no idealization is allowed. The assembly cannot be critical—of tolerance 0%—, ie two precisely identical resistors cannot be used.) Extra question: can this bomb be actually built without the builder being killed? Re-engineer the process so that it should be accomplishable. :)

   (András Pál)

2. A small body is sliding on an inclined plane of angle α. The coefficient of kinetic friction μ is larger than $\tan\alpha$, thus the body set into motion in an arbitrary direction will come to a halt after a while. How does the direction of the body's velocity immediately before stopping depend on the body's initial direction and speed?

   (Dezső Varga)

3. A long thread is wound around a cylinder of mass M whose axis is horizontal. A mass m is hung to the end of the long thread (much like a bucket onto the chain of a draw-well). When does the thread get unwound faster, if the weight

   a) moves perfectly along a vertical line,

   b) performs small sidewise oscillations while moving roughly along a vertical line?

   The thread is tightly wound around the cylinder, and its top part of length l_0 is prevented from sidewise motion (with a suitable bracket). Friction, thread weight, and resistance of the medium can all be neglected.

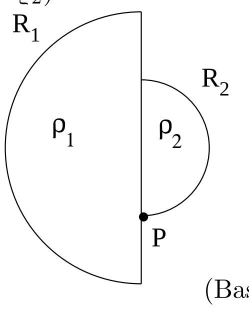
   (Péter Gnädig)

4. In a circus stunt a large, solid cylinder that is corrugated on its outside is hung up by very long and thin threads to the ceiling high above. Another similar cylinder is placed on top of the first one, paralelly to it. The two cylinders are of equal mass but different radius. The system gets displaced from its unstable equilibrium position, and the top cylinder will sooner or later fall off the top of the bottom one. (The corrugated surfaces prevent the two cylinders from sliding on one another but do not obstruct their motion otherwise.) Determine the angle between the vertical direction and the plane containing the two cylinder axes in the moment of detachment.

   (Péter Gnädig)

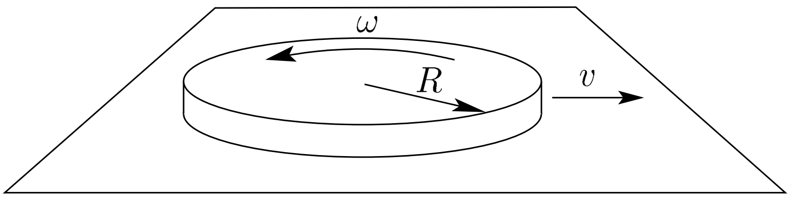
5. On one of his missions, Inspector Gadget discovers a huge UFO, which is hovering in empty space and is consisted of two ginormous hemispheres—supposedly of uniform mass distribution—kept together by gravitational forces. I. Gadget finds a small slit in point P, and tries to push the two hemispheres apart there. What force does he need to exert?

   Chief Quimby (hitherto disguised as a rather odd looking garbage bin so as to keep an eye on that fool Gadget) ascertains upon inspection that the radii of the two hemispheres are R_1 and R_2. By means of chemical analysis, I. Gadget determines their densities (ϱ_1 and ϱ_2).



   (Based on a problem of Gagik Grigoryan, Armenia)

6. I happened to push a compact disk lying on a table: it started sliding as well as spinning (see figure). The disk, face down throughout the motion, was moving along a straight line, and came to a halt in a while. I was surprised to see that the two kinds of motion (translation and spinning) stopped simultaneously. I first thought this must be a rare coincidence due to the initial conditions. I was wrong! I found out that no matter how the CD is pushed, translation and spinning stop simultaneously. That is, barring the two trivial cases (in which the disk performs pure translation or pure spinning initially), the disk never slides without spinning, or never spins without sliding. Give it a go yourself, and try to explain the phenomenon.



   Note: It is important that the disk and the sliding surface (table, floor, carpet) should be perfectly level. Small friction is advantageous for the experiments (as the motion does not stop so fast). Sliding the disk on a carpet proved to be the best choice in my experiments.

   (Salamon Zorge)

7. Describe the classical motion of two electrons in a magnetic field. Study the stability of the orbits.

(Zoltán Kaufmann and József Cserti)

8. The Hamiltonian of a particle moving in three dimensions is

$$H(\mathbf{r}, \mathbf{p}) = pc(\mathbf{r}) + U(\mathbf{r}),$$

where $p = |\mathbf{p}|$, and $c(\mathbf{r}) > 0$ and $U(\mathbf{r})$ are given functions of the position.

a) Write up Hamilton's canonical equations of motion, and express therefrom the particle's acceleration vector $\mathbf{a} = \ddot{\mathbf{r}}$. Simplify this equation through equivalent steps. What is the meaning of $c(\mathbf{r})$ and of $U(\mathbf{r})$?

b) Calculate the angular momentum and the particle mass.

c) Calculate the particle's Lagrangian, and try to re-derive the Hamiltonian therefrom. Did you tumble across something odd? How could the problem be circumvented, ie how could one derive the equations of motion from the Lagrangian formulation in spite of this oddity?

d) Discuss two special cases: when the function $c(\mathbf{r})$, and when the function $U(\mathbf{r})$ is constant. What will the motion be like if these functions are constant only in the two halves of space, with a finite jump across the boundary? What if i) either function $c(\mathbf{r})$ or $U(\mathbf{r})$, ii) both functions make a jump across the boundary?

e) Study the special case when functions $U(\mathbf{r})$ and $c(\mathbf{r})$ are functions of the radial distance $r = |\mathbf{r}|$ from the origin. Under what conditions does a circular orbit of radius R exist? What about the stability of the circular orbit? Write up the differential scattering cross section for particles coming in from infinity.

f) Extra question: What does this problem have to do with Alice, Monsieur Fermat, and Herr Einstein?

(Gyula Dávid)

9. Consider n particles in a hyperbolic plane moving in each others' potential fields. What kind of mechanics can be constructed assuming that it is locally identical to the well-known classical mechanics of Euclidean space? What are the symmetries, and what are the counterparts of linear and angular momentum conservation? How are angular momenta referred to different points related? Analyze the applicable forms of Newton's law of gravitation and of the two-dimensional harmonic oscillator.

(Balázs Pozsgay)

10. Point-like bodies of unit mass are connected by springs of unit length and unit spring constant, and placed along an infinite half-line. The system is initially at rest. The outermost body suddenly starts to be pulled at a uniform rate. How does the Nth element of the chain move? Examine the continuum limit as well.

(Péter Gnädig)

11. The stress at point $\mathbf{r}$ of a non-local elastic continuum depends not only on the strain at point $\mathbf{r}$ but also on the strain at point $\mathbf{r}'$:

$$\sigma_{ik}(\mathbf{r}) = \int_V C_{iklm}(\mathbf{r} - \mathbf{r}')\varepsilon_{lm}(\mathbf{r}')dV',$$

where σ_{ik} is the non-local stress tensor, ε_{lm} is the well-known strain tensor, V is the medium's volume (chosen as infinity in the present problem), and finally C_{iklm} is the tensor describing the non-local interaction (according to the generalization of Hooke's law), which takes the following form in a homogeneous, isotropic medium:

$$C_{iklm}(\mathbf{r} - \mathbf{r}') = \lambda_0 \alpha(\mathbf{r} - \mathbf{r}')\delta_{ik}\delta_{lm} + \mu_0 \beta(\mathbf{r} - \mathbf{r}')(\delta_{il}\delta_{km} + \delta_{im}\delta_{kl}),$$

where λ_0 and μ_0 are the Lame constants used in the long wavelength approximation (for usual local continua); $\alpha(\mathbf{r})$ and $\beta(\mathbf{r})$ are unknown functions. In the absence of body forces, the equation of motion for the continuum of mass density ϱ is of the usual form

$$\partial_j \sigma_{ij} = \varrho\, \partial_t^2 u_i.$$

Show that if the dispersion relations of longitudinal and transverse waves propagating in the medium are known, then it is possible to determine the Fourier transforms of the functions $\alpha(\mathbf{r})$ and $\beta(\mathbf{r})$. Consider dispersion relations for longitudinal and transverse waves of the form

$$\omega^2 = \begin{cases} \frac{c^2k^2}{D(k^2)}, & \text{if } |\mathbf{k}| \le k_B, \\ 0, & \text{if } |\mathbf{k}| > k_B, \end{cases}$$

where $c = c_L$ and c_T are the long wave longitudinal and transverse sound speeds, $D = D_L$ and D_T are given functions characteristic of the dispersion, and k_B denotes the edge of the Brillouin zone.

(György Vörös)

12. A small circular hole is cut in the middle of a drum. How do the eigenfrequencies change—assuming that the membrane does not break? What if the hole is cut off-center?

(József Cserti)

13. How does the fundamental tone of a column of air within a tube change upon opening a small hole on the side of the tube? In other words: how does the pitch of a pipe change, if the musician lifts their fingers, or that of a flute, if the keys are pressed? Before the brute-force numerical solution try to find a perturbative method for calculating the frequency shift.

(András Vukics)

14. Imagine a candle that fills up the whole half-space $y \leq 0$, and whose wick is the whole half-axis $y \leq 0$. What will be the shape of the crater in the wax after "infinitely" long time?

(Gábor Veres)

15. Frozen meat of initial temperature $-20°$ C is warmed up in a microwave oven, using 600 watts. Determine the temperature vs. time function. The mass of the meat is m, and it contains 40% of dry material. The specific heat of dry meat is c.

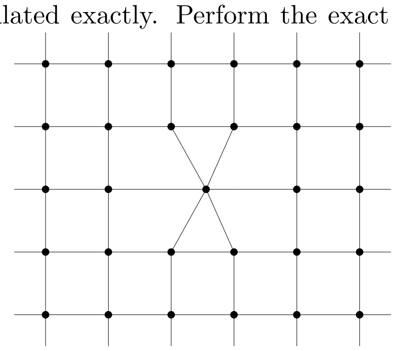
Consider first the case when steam does not escape from the oven, and second, when pressure is constant within the oven.

(Péter Pollner)

16. In a diffusion cloud chamber alcohol vapor is sinking in the air of inhomogeneous temperature. The side of the chamber is a straight cylinder made of transparent insulating material (if it were not transparent nobody could see anything). The bottom is a good heat-conducting metal cooled usually down to $-20°$C. The top of the chamber can also be heat-conducting—but this is often unimportant. Ionizing radiation can produce trails in the overcooled area. The larger the temperature gradient in the bottom part of the cylinder, the more visible the trails are. Design a cloud chamber and determine the vertical temperature profile inside (using an adequate approximation). Think about the geometrical properties such as height and radius, including those of the insulating wall—thickness, shape, etc—as well as the feasibility of the whole construction.

(Ákos Horváth)

17. An infinite square lattice is made up of identical resistances R_0, then an edge is removed, and the two nodes of lower degree are short-circuited. Determine the resultant resistance between any two points of the network, or at least demonstrate that it can be calculated exactly. Perform the exact calculation for some pairs of points.



(Előd Gáspár Merse)

18. Two grounded half-planes, having a common edge, make an angle α with each other. A uniformly charged rod of length L and charge Q is placed in the angle bisector plane, parallelly to and at a distance r from the the common edge of the two planes ($r \ll L$).

a) Determine the force acting on the rod if π is an integral multiple of α.

b) Determine the force acting on the rod for other angles α.

c) How is the result affected if the rod is placed outside the angle bisector plane?

(Péter Gnädig)

19. A short-circuited superconducting coil (number of turns N_1, length ℓ_1, radius R_1) is affixed to a long, vertical glass tube. Another short-circuited superconducting coil (mass m, number of turns N_2, length ℓ_2, radius R_2) is dropped into the glass tube. There is just enough room for this second coil in the tube. Initially, current I_0 is circulating in the first coil; no current is flowing in the second (moving) coil.

Examine the motion of the second coil within the tube. (Assume $\ell_{1,2} \gg R_1 \approx R_2$. Friction and resistance of the medium can be neglected.)

(Péter Gnädig)

20. Ányos Jedlik's (1800-1895, the Hungarian inventor of the dynamo) "dividing machine" could produce not only straight-line optical gratings but also so-called circular gratings. This latter is a plane spiral of constant pitch, which, to a good approximation, can be viewed as a uniform-density sequence of concentric circles. The machine was later perfected by Jedlik's student and fellow member of the Benedictine order, Gergely Palatin (1851-1927). Suppose that a Palatin-type circular grating (containing 77 lines per millimeter along the radius) and a straight-line optical grating (containing 150 lines per millimeter) are superposed. Determine the resulting diffraction pattern. Experimentally, one can proceed like this: The image of a tiny hole illuminated by a bulb is formed on a screen. The two gratings are placed along the light path in front of / behind the imaging objective. What can be seen on the screen? Why?

(Radnai Gyula)

21. A polarizer is placed into each arm of a Mach-Zender interferometer (see, eg, M. Born, E. Wolf: Principles of Optics). How does the visibility of the interference pattern depend on the angle made by the polarization directions? How does the visibility of the pattern change, if a third polarizer of adjustable polarization direction is placed into the path of the exiting light? The experimental set-up can be studied, and the experiment can be virtually performed with a program downloadable from

`http://www.physik.uni-muenchen.de/sektion/didaktik/Computer/interfer/Interferometer.zip`

(Mihály Benedict)

22. In all textbooks one arm of the Michelson-Morley interferometer points in the direction of the supposed ether wind, and after a first measurement the apparatus is always turned through 90 degrees. Show that the Lorentz contraction hypothesis gives account of the null result of the Michelson-Morley experiment, whatever angle the directions of interferometer arms make with that of the ether wind.

    Extra question: what stress is created by the Lorentz contraction in the interferometer arms (right cylinders or right square prisms made of homogeneous and isotropic material)?

(Gyula Dávid)

23. According to the special theory of relativity, traveling forward in time is possible, if one leaves the vicinity of the earth (considered stationary), accelerates, reaches almost to the speed of light, and then returns.

    a) Plan a journey to visit your twelfth-generation descendant, who lives 300 years hence.

    b) Give the shape of a closed loop along which one should travel in space if one wishes to grow older as little as possible by the end of the journey to the twelfth-generation descendant.

    c) How much proper time is minimally needed if one wishes to see the universe twice as old as now?

    In each case one should return to and stop in the vicinity of the earth (ie. slow down to non-relativistic velocities). Do not forget about the tolerance limitations of the human body: the experienced acceleration should not ever exceed g, ie the acceleration due to the earth's gravity.

(Bence Kocsis)

24. Study the relativistic Kepler problem, ie the motion of a particle at speeds comparable to that of light in a fixed central potential field $V(r) = -\alpha/r$. Determine the orbital equation of the bound particle. Calculate the angle of perihelion precession. (Determine the part of Mercury's perihelion precession due to this effect.) What trajectory is traced out by a particle launched at infinity? Calculate the differential cross section of scattering, and compare it with its non-relativistic counterpart.

    Tip: The calculation can be facilitated by using the parameter

$$\gamma = \arcsin\left(\frac{\alpha\sqrt{1-v^2/c^2}}{mr^2\omega c}\right),$$

    where c is the speed of light, m is the rest mass of the moving particle, r is its distance from the center, v is its (ordinary, three-)velocity, and ω is its angular velocity. Show that the parameter γ is a constant of motion, thus it suffices to calculate γ in the initial moment. What could be the geometrical meaning of parameter γ?

(Gyula Dávid)

25. Interstellar material consists predominantly of hydrogen. One would like to build a spacecraft that would accelerate by burning in its fusion reactor the hydrogen (collected by means of a "funnel") as the spacecraft moves through the interstellar hydrogen. Determine the (ultra)relativistic resistance of the medium due to the collection of hydrogen. How does this influence the maximum attainable speed of the spacecraft (if such exists)? How long would it take for this spacecraft to reach the Andromeda Galaxy? Assume the density of interstellar medium to be uniform, and of the *real* magnitude of 1 hydrogen atom per cubic meter.

(András Pál)

26. In the past decades tremendous amounts of money were spent world-wide on the building of gravitational wave detectors. The newly built LIGO (Laser Interferometer Gravitational Wave Observatory, USA) finished its first measurement period last month, however, according to the preliminary disclosure of results, no gravitational wave event has been found. Only signals emitted by the most violent astrophysical sources are expected to exceed the noise limit.

    Give an upper limit on the maximum energy emitted in the gravitational wave channel during a supernova explosion. How large must a 100% efficiency interferometer be (at least) in order to detect such a signal?

(Bence Kocsis)

27. According to Hawking, black holes emit thermal radiation, the temperature of which is inversely proportional to the mass of the black hole. Study how the temperature and the mass change with time for a black hole (initially of 5 solar masses, created these days). Account should be taken of the fact that the black hole is not an isolated object but one floating in (and in thermal equilibrium with) the "heat reservoir" of cosmic background radiation. The temperature of cosmic background radiation (presently 2.7 K) varies as the $-2/3$th power of the time elapsed since the Big Bang. How long does a black hole live? What is its scream of death like?

(Gyula Dávid)

28. Quantize the system in Problem 7.

(József Cserti)

29. Within the framework of non-relativistic quantum mechanics, study the motion of a charged particle in the field of a point-like electric dipole. Under what condition(s) do bound state exist?

(Egri Győző)

30. The Schrödinger-Kostin equation, proposed for the quantum mechanical description of velocity-proportional damping reads

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\Delta\psi + U\psi - ik\frac{\hbar}{2}\psi\ln\frac{\psi}{\psi^*},$$

    where k is the damping factor. Justify why this equation can be regarded as the quantum mechanical equation of motion for a particle moving in potential U and damped by a "drag force" proportional to the velocity. Determine the stationary states. Show that energy is dissipated.

(Péter Ván)

31. In semiconductors, electrons and holes are created in pairs by thermal excitation, and annihilated upon recombination. This process is described by the chemical reaction

$$e + h \rightleftharpoons 0$$

    which yields equilibrium. In equilibrium, the concentrations of electrons and holes depend on temperature T according to the Arrhenius equation

$$n_e = N_e e^{-E_1/kT}, \;\; n_h = N_h e^{-E_2/kT},$$

    in which E_1 and E_2 are given energies. Determine the reaction constant and the reaction heat of the above process from the thermodynamical relations concerning chemical reactions.

(Géza Tichy)

32. A car is approaching a traffic light. The red light is noticed at $t = 0$, from distance l (which can be taken as unity), while traveling at speed V. The car can accelerate with A, and decelarate with $-A$ (for the sake of simplicity; this simplification can be generalized). The traffic light turns green with a probability distribution $P(t)$. Consider three cases:

    1) $P(t)$ is a Dirac delta function $\delta(t - T)$, ie it is known for sure that the light will change at $t = T$. (One is traveling on a known route, and knows when the traffic lights change relative to each other).

    2) $P(t)$ is a uniform distribution in the interval $[0, T]$, ie the light changes any time between the present moment $t = 0$ and $t = T$. (One is traveling on a known route but does not know when the light had changed).

    3) $P(t)$ is an exponential distribution proportional to $\exp(-t/T)$. (One is traveling on an unknown route and can only guess the behaviour of the traffic lights).

    Define 3 target functions:

    a) Driver in a hurry: The expected time of reaching a point at a large distance L ($L >> l$) beyond the traffic light should be minimized.

    b) Driver in a hurry but obeying the speed limits: The same as a), but the speed of the car cannot exceed $V_{\max}$.

    c) Energy saving driver: The driver wants to slow down as little as possible, ie the minimum speed should be as high as possible. One more constraint: the driver is not to accelerate before the light changes to green.

    What is the optimum driving strategy for the 9 possible combinations of the 3 target functions and the 3 probability distributions?

(Gábor Tóth)

33. Construct a portfolio.

Suppose that one can choose among three forms of investment: stocks, bonds and money market. Constructing a portfolio means making a decision about how to share one's money among the different forms of investments, and about the term one lets the portfolio run. Follow the "buy and hold" strategy, ie keeping the portfolio for the determined amount of time and withdrawing the profit (or compensating the losses). For simplicity, neglect inflation and tax on the profit. (Ahem.)

Assume that the annual rate of return of the investment forms has a trivariate Gaussian distribution with the following phenomenological parameters.

Expectation and standard deviation of annual rates of return:

	expectation	standard deviation
stock	10%	15%
bond	5%	6%
mm	3%	1%

Experience shows that different rates of return are correlated in a given year. The correlation of variables x and y is defined by

$$C(x,y) = \frac{< (x- < x >)(y- < y >) >}{\sigma(x)\sigma(y)}.$$

In an economical environment favoring the exchange of stocks, the rate of return is usually smaller for bonds and money market. Use the following parameters for the correlation:

stock – bond	−0.3
stock – mm	−0.1
bond – mm	0.2

Assume that the probability distribution of rates of return stays the same over the years, and that the rates of return are independent (uncorrelated) in individual years.

a) Determine the expected rate of return and its standard deviation for a portfolio of arbitrary composition and arbitrary term.

b) The risk of a portfolio means the probability that in the given (full) term the portfolio does not make profit (but it makes losses instead). Propose portfolio combinations that maximize the expected rate of return when the term (1, 2, 5, 10, 30 years) and the risk (1%, 2%, 5%) are fixed.

c) Give a rationale for the investors' rule of thumb: Diversification above all! If one tries to save for a car, stocks should be avoided, while one should invest into them when trying to save for an extra old-age pension.

(Zsolt Bihary)

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