THE 40th—12th INTERNATIONAL—
RUDOLF ORTVAY
PROBLEM SOLVING CONTEST IN PHYSICS
2009

The Physics Students' Association of Eötvös University, Budapest, the Roland Eötvös Physical Society and the Hungarian Association of Physics Students proudly announce the **40th** – and for the twelfth time **international** Rudolf Ortvay Problem Solving Contest in Physics, between 22 October 2009 and 2 November 2009.

Every university student from any country can participate in the Ortvay Contest. PhD students compete in a separate category. The contest is for individuals: solutions sent by groups of students are not accepted. The name, the university, the major, and the university year should be indicated on the solutions. Pseudonyms and passwords cannot be used: each contestant has to use his/her own name.

The problems can be **downloaded** from the webpages of the Ortvay Contest

http://ortvay.elte.hu/

in Hungarian and English languages, in pdf format, from **12 o'clock (Central European Time, 10:00 GMT), Wendesday, 22 October 2009**. The problems will also be distributed by local organizers at many universities outside of Hungary.

Despite all the efforts of the organizers, it may happen that some unclear points or misprints stay in the text. Therefore it is very useful to visit the webpage of the contest from time to time, as the corrections and/or modifications will appear there.

Each contestant can send solutions for up to 10 problems. For the solution of each problem 100 points can be given.

Any kind of reference material may be consulted; textbooks and articles of journals can be cited.

Each problem should be presented on (a) separate A4, or letter-sized sheet(s). The contestants are kindly asked to use only one side of each sheet. Solutions written by pencil or written on thin copy-paper will not be accepted as these cannot be faxed to the referees.

Computer programs appended to the solutions should be accompanied by detailed descriptions (what computer language it has been written in, how to use the program, which parameters can be set, what notations are used, how to interpret the output figures and graphs, etc.) They can be enclosed on floppy disks, or sent via email to the addresses below.

Solutions can be sent by mail, fax, or email (in LaTeX, TeX, pdf or Postscript formats). Contestants are asked not to use very special LaTeX style files unless included in the sent file(s). Electronically submitted solutions should be accompanied—in a separate e-mail—by the contents and, if necessary, a description explaining how to open it.

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H-1117 Budapest, Pázmány Péter sétány 1/A, HUNGARY
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Deadline for sending the solutions: 12 o'clock CET (11:00 GMT), 3 November 2009.

Contestants are asked to fill in the form available on our webpage after posting their solutions. It will be used for identification of contestants and their solutions. **Without filling in the form, the organizers cannot accept the solutions! The form is available only on 2 to 3 November.**

The contest will be evaluated separately for each university year, according to the total number of points. The referees reserve the right to withhold, to multiply or to share some prizes. Beyond the money prizes given for the first, second, and third places, honorable mentions and special prizes for the outstanding solutions of individual problems can be awarded. This is why it is worthwhile sending even one or two solutions.

The announcement of the results will take place on 10 December, 2009. The detailed results will be available on the webpage of the contest thereafter. Certificates and prizes will be sent by mail.

Wishing a successful contest to all our participants,

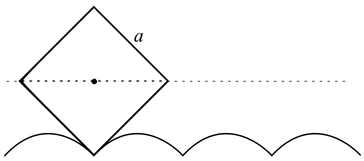
the Organizing Committee: Gyula Dávid and József Cserti
The original Hungarian text is translated by Dezső Varga and Gábor Takács

1. In the middle ages it was believed that Earth is flat. Ancient Greeks however, were aware of the fact that it is rounded! They even determined its radius. As it is known, 'Eratosthenes measured the circumference of the earth without leaving Egypt. Eratosthenes knew that on the summer solstice at local noon in the Ancient Egyptian city of Swenet (known in Greek as Syene, and in the modern day as Aswan) on the Tropic of Cancer, the sun would appear at the zenith, directly overhead. He also knew, from measurement, that in his hometown of Alexandria, the angle of elevation of the Sun would be 1/50 of a full circle (7°12') south of the zenith at the same time. Assuming that Alexandria was due north of Syene he concluded that the distance from Alexandria to Syene must be 1/50 of the total circumference of the Earth.' (wikipedia.org)

   Now what if Earth is really flat? If the Sun is at a finite height above the surface, then if it is exactly vertically above one city, in an other city the rays will be included still. How could the Greeks be able to decide which explanation is correct? Let us attempt with measuring the angle of the incident sunlight within a limited geographical region! With which precision should one perform the angular measurement to clear up the question?

   (Zoltán Kaufmann)

2. A cube with homogeneous mass distribution, of side length of a, can 'roll' over a well designed surface without slipping, such that its center moves along a horizontal straight line ('square-shaped wheel').
   a) What is the shape of the surface?
   b) If this special 'passage' is inclined at an angle of α, the cube will roll along without slipping (friction is large enough). Is it worthwhile to change over from the traditional, circular-shaped wheel, if we aim at the fastest rolling down the slope?



   (Máté Vigh)

3. We try to 'swallow' spaghetti (i.e. long, cylinder-shaped compact pasta). The pasta evidently moves into our mouth, because the pressure in the mouth cavity decreases. The force exerted by our mouth is on one hand only pressing force (this is why air does not floats in as well), and on the other hand friction by the lips, which exerts force outwards. So then what presses inside? The spaghetti is cylindrical all the way along, thus external air pressure is not pressing 'inwards', but perpendicularly. We may say, that pressure is exerted at the ends: but who would think that if something presses the far end of the meter long pasta on the plate, anything happens? We do not have the feeling either that something would 'pull' the other end of the pasta, from the inside of our mouth. Let us give the forces which are exerted on the pasta, and show that the pressing force towards the inside depends only on the pressure difference and the cross sectional area of the pasta.

   (Dezső Varga)

4. A coil spring of mass m, length l and spring-constant D is at rest on a flat horizontal surface. One end of it is fixed, the other is hit by a block of mass M moving with a speed v along the axis of the spring. The collision is elastic.
   a) After what delay and at what a speed does the block bounce back, if m is significantly but not negligibly smaller than M?
   b) What happens if the mass ratio is reversed?

   (Ferenc Woynarovich)

5. A pointlike body is thrown vertically upwards. Initially the mass of the body is m_0, and it varies during the flight in the following way, depending on the time t after the instance of throwing:

$$m(t) = m_0\, e^{\frac{t}{\tau}},$$

   where τ is a constant with dimension of time. The body can be considered to be pointlike all the time, and the increment of mass moves together with the body. The initial velocity of the body is $v_0 = g\tau$ started from the surface of Earth.
   a) After how much time does the body get to the climax point of orbit?
   b) What is the mass at this point? (The rotation of Earth is negligible).

   (Bonbien Varga)

6. Consider a one dimensional harmonic potential complemented with a power series beginning at a cubic term. How should one choose the all nonzero Taylor coefficients so that the exact period becomes independent of the amplitude? Try to guess the full potential function either from a few lower order coefficients, or, from other simple physical reasoning, and then show the constancy of the period.

(Géza Györgyi)

7. Let us assume that an 'extra' dimension exists, with extension d and with periodic boundary conditions, that is, the topology of space is form of $\mathbb{R}^3 \times S_1$ with the usual flat Eucledian metric. Determine in this space the gravity potential of a pointlike mass! (Hints: the exact result can be given with a simple formula; considering the Euler-type solution of the Basel-problem may be helpful). Let us show, that for very long and very short distances the usual three or four dimensional Newtonian potential is restored! How could this problem be solved in the case of $\mathbb{R}^5 \times S_1$ or in general, in case of $\mathbb{R}^{2n+1} \times S_1$?

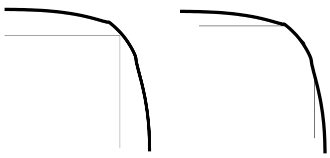
(Balázs Pozsgay)

8. An unstretchable rope of length L, with negligible mass, is fixed at one end to the ceiling such that its lower end can move freely (in homogeneous gravitational field). Let us fix on the rope n small balls, all of mass m, at equal distances, each pair separated by $l = L/n$. The first ball is attached lower than the fixing point of the rope by l; the last, n-th ball is fixed at the other end of the rope. Let us assume that all balls are moving in a common plane. The system is a generalization of the mathematical pendulum, let us call it 'n-pendulum'.
a) Give the eigen-frequencies of the small amplitude oscillations of the n-pendulum, and the corresponding normal modes! Plot graphically the first few oscillating modes for different values of n!
b) Calculate the sum of the inverse of the squared eigenfrequencies! What interesting result can be noted?
c) Study in detail the $n \to \infty$ limiting case! Calculate the eigenfrequencies in this limit! Let us check the result by direct calculation, that is, determine the eigenfrequencies of the small amplitude oscillations for a rope of length L with homogeneous density distribution, which is fixed at one end. Do we get the same result? If not, then why not?

   Bonus question: what surprising mathematical relation can be achieved, if we compare the result of part b with part c?

(Bonbien Varga)

9. A rope is placed over an edge, according to the figure on the left side. The rope is very long, weightless and very thin; however, it is 'stiff', that is, in case of bending, its radius of curvature is inversely proportional to the torque appearing on the rope. The rope may slip over the edge without friction. The rope is pulled with a given force horizontally towards the left, and also vertically downwards.
a) Give the differential equation which determines the shape of the rope (e.g. using the parametrization along the length of the rope, or in any other convenient form!) Determine the function describing the shape of the rope!
b) The rope is pulled many times over the hard edge. The edge starts to wear away slowly (however, the friction is still negligible from the point of view of the shape). After a while the sharp edge takes the shape similar to the figure on the right side. Let us assume that the speed of wearing is proportional to the force with which the rope is pressing the material of the edge. What will be the shape of the worn edge after some time? Let us assume two extreme cases: what if the rope is extremely 'soft' or it is extremely 'stiff'? The general case may be attempted to determine in a numerical fashion.



(Dezső Varga)

10. A high speed meteorite hits a spherical shaped, atmosphere free asteroid. From the point of impact, rock debris is emitted to space, part of which falls back on the surface after a while. Let us assume that all emitted rock pieces are of the same size, and also that their direction of emergence is uniform over the upper hemisphere. Let us disregard the secondary effects of the debris falling back down, or rebouncing: the rock pieces which reach the surface will remain there.
a) Make a further assumption, that all emitted pieces of debris are of the same initial velocity. Determine the distribution of the fallen debris layer over the asteroid's surface, and present the results as a function of the initial velocity! Which fraction of the total emitted rock fall back on the surface?
b) Let us assume that the angular distribution is uniform, but make no assumption on a common initial velocity! Can there be such a distribution of velocities, which results in a uniform thickness of fallen debris over the whole surface of the asteroid?

(Gyula Dávid)

11. Water is poured out of a bottle. The water flows out, then starts to dribble: in this later phase, the speed of the dribbling is reduced by the finite viscosity of the water which 'sticks' on the bottle wall. Let us consider a simpler situation, when the water is poured off from a slightly inclined planar surface. Let us assume that the surface tension and the evaporation is negligibly small. What is the (continuously slowing) speed of the water? Let us try to investigate the phenomenon also experimentally, to check if the predicted time dependence is realized indeed (from the frequency of the dribbling, the flow velocity of the water can be extracted). Additional questions to investigate: if the pouring is performed from some practical container, to what extent do the qualitative result depend on the shape of the container? Let us try to answer the frustrating question: how much tea is remaining in the teapot (or beer in the bottle, whoever is frustrated more by which case) if it dribbles only very slow?

(Dezső Varga)

12. What is the direction of the motion of the middle of the smoke ring on the picture? Why?

(Máté Vigh)

13. There are two glasses, both at 20 degrees Celsius, both filled with 0.3 l of water. Into one of them we put 0.05 kg of salt, also at 20 degrees, and wait until it dissolves. After that in each glasses we place an icecube of -10 degrees, weighting 0.1 kg. In which glass will the ice melt earlier, and approximately how much earlier, and why? Confirm your qualitative answer with calculation!

(Máté Csanád)

14. Since Einstein (and Smoluchowski) we know and learn, that if we know the drag coefficient γ of a small particle (measured or calculated), then measuring the diffusion coefficient D caused by the Brownian motion, the temperature of the medium is given by the following simple formula: $k_{\mathrm{B}}T = \gamma D$. Let us consider a spherical Brown-particle moving in low pressure gas, where it hits only a single gas molecule at a time. Let us assume that we are able to detect every single collision of the Brown-particle, that is, we know the series of velocities after each collisions, $V_1, V_2, \ldots, V_n$. Knowing only this (with the Brown-particle of radius R, mass M and also the total time T of the measurement given) let us determine the pressure p of the low pressure gas medium!

(Lajos Diósi)

15. Let us consider the following linear, stochastic differential equation system:

$$\frac{d\hat{\mathbf{v}}(t)}{dt} = \mathbf{M}\hat{\mathbf{v}}(t) + \hat{\mathbf{q}}(t),$$

where $\mathbf{M}$ is an arbitrary square matrix, with known λ_k eigenvalues and $\mathbf{l}^{(k)^*}$ and $\mathbf{r}^{(k)}$ left and right eigenvectors, respectively. Besides, $\hat{\mathbf{q}}$ is a white noise, which satisfies the following correlation relation between its components:

$$\langle \hat{q}_i\,(t')\,\hat{q}_j\,(t'')\rangle = D_{ij}\,\delta\,(t' - t'')\,.$$

Let us study the time dependence of correlation between the components of $\hat{\mathbf{v}}$: provided that $\langle \hat{v}_i\,(0)\,\hat{v}_j\,(0)\rangle$ is given, determine $\langle \hat{v}_k\,(t)\,\hat{v}_l\,(t)\rangle$!

(Gábor Kónya)

16. On the website *http://www.pokerstars.hu/poker/room/features/security/* a summary can be read related to problems of algorithm of evenhanded shuffle. Here a shuffle algorithm can be found which is said to be reliable and is free of 'bad distribution of shuffle'. Let be given deck of N cards which is mixed by the algorithm found on this website! Let investigate, how many cards remain in its original place after mixing. What is the probability, $P_{N,k}$ of that exactly $0 \leq k \leq N$ cards remained in its original place? What is the expected value and standard deviation of k for values of N? Calculate the entropy of shuffle, $S_{(N)} = -\sum_{k=0}^{N} P_{N,k} log_2 P_{N,k}$! Give the value of $P_{N,0}$ in the case of $N \to \infty$ and estimate the difference between $P_{N,0}$ and this limit!

(Gábor Homa and Krisztián Kis-Szabó)

17. In 1873, in the Hungarian Academy of Science, the 25 year old Loránd Eötvös has presented the essay of the 20 year old Izidor Fröhlich, with the following words: 'Izidor Fröhlich third year student has shown, that based on the theory of atmospheric refraction, the spherical celestial objects (Sun, Moon) must appear elliptical, if the elevation angle of the celestial object is not less than 5 degrees'. Let us investigate in which approximation can this statement be true.

(Gyula Radnai)

18. A layer with parallel walls and of thickness of d has a given varying refraction index $n(z)$, where z is the coordinate perpendicular to the surface of the plate. Light is incident perpendicularly on one side of the layer. Which fraction of the light can pass through the plate? Let us assume that the refraction index is a slowly varying function of z, that is, $|dn/dz| \ll 1/\lambda$, where λ is the wavelength of the incident light. Let us investigate in detail relevant and interesting special cases!

(József Cserti)

19. Superconducting material can float over a permanent magnet, due to the fact that its magnetic moment changes in a specific way depending on the external magnetic field (for the superconductor, this dependence is such that the magnetic field vanishes inside). Let us consider now the other limiting case, that is, when a permanent magnet(system) is floating above permanent magnets. Permanent magnet is defined by the property that its magnetic moment is independent on the outside magnetic field strength, however this magnetic moment density can vary inside the magnet depending on the position. Is it possible that an arbitrary system of permanent magnets (including the possibility of being connected by solid rods) can float above properly placed permanent magnets?

(Dezső Varga)

20. Dr. Albert Einundzwanzigstein was teaching special relativity (1+1 dimensional only) for many years now to the students of the University of Faraway – with less and less success recently. First his formulae was not understood, then his theoretical expressions were received with uneasiness. The trouble became serious when he was talking about the relativity of simultaneity (i.e. if two events happen at the same time in one frame, they may not be so at a moving reference frame). As we all know, the recently introduced ZYXSc educational system states in its Third Guideline Policy that the loudness of the students is inversely proportional to their knowledge, then Albert was not totally surprised when one of his nicest students, Romeo Butt stood up after class and said so: Listen old man! If Juliet and myself get to the climax at the same time, that this is undoubtedly simultaneous! No chatting about relativity stuff, and if you would like (is it so?) to send an observer I will break his neck! Am I clear? (Romeo Butt wanted to add originally, that 'Eight inches is surely eight inches, no more, no less, you got that?' but Juliet and her girlfriends efficiently made him withdraw this.) Albert looked up at the two-feet taller Butt, and nodded sadly. When Albert told about this story to the Dean at the Faraway University, he agreed actually with the students: 'What did you expect? They are paying, we are only servicing. Why do you insist on your outdated theories? I must tell you as well that the guys were complaining that you write the table full of some weird pointed and squarry signs which nobody could understand at all ...' Albert tried to figure out this latter issue, and he realized that the operation of taking the square-root has not been taught any more at the secondary schools of Faraway, therefore no wonder that his students could not understand the usual transformational formulae. On the advice of his friend, Lőrinc, he attempted at reformulating the theory. To his surprise he was very soon successful. His new theory (which he called re(duce)lativity specially to be presented at the XYZSc courses) was based on the following basic assumptions: 1) Absolute time, events which are simultaneous in one frame are also simultaneous in any other reference frames, $t' = t$ (the guy was really two feet taller ...). 2) Universality of the speed of light: if in the reference system K $dx = c\,dt$, then it follows, that it is also true in the reference system K': $dx' = c\,dt'$ (here c is a physical constant of dimension of velocity). 3) Mathematical simplicity of the formulae: In the formulae, no square-root, hyperbolic functions or any exotic operations can appear, only the four basic operations. The results of the calculations were finally expressed in a presentable form by his friend Lawrence, this is why they are called now the 'Lawrence transformations'. Lawrence finally came up with a physics formula: the easiest way to remember them is if we assume that the extension of any object in direction x and moving in direction x at a speed of V will be multiplied with a factor depending on V/c (so called Lawrence contraction).

The end of the story is a sad one. For a few years the re(duce)lativity theory was taught happily, when the next reform of the education at secondary school level canceled the teaching of the operation of division. Albert introduced the $c = 1$ unit system, that is, one could write V instead of V/c. However after few later years the last reform completely abandoned the subtraction and the brackets, dr. Albert Einundzwanzigstein totally gave it up. Presently he is selling popcorn in the adult cinema of Faraway, whereas Lawrence is experimenting on artificially tasted melon retail. When being asked about the re(duce)lativity, they both keep deeply quiet.

In an attempt to reveal the actual historical facts, evidently Romeo Butt's lecture notes seemed to be a possible source, but what was found there instead of actual lecture notes was something unquotable. As to now, seemingly the memorable Lawrence transformation formulae are lost forever.

Or maybe not? Maybe the participants of the Fortieth Ortvay Competition can reveal this forgotten beauty of scientific history, based on the above known facts. An attempt at least is worthwhile...

a) Write down the 1+1 dimensional Lawrence transformational formulae corresponding to the three above mentioned criteria in the form of $x' = f(x, t, V)$. The reference frame K' is moving towards the left at speed V seen from the frame K.
b) Study the inverse transformation! What is the apparent speed V' of frame K seen from the reference frame K'?
c) Is there a maximal velocity existing, and if so, at which value?
d) Write down the function which appears in the Lawrence contraction formula!
e) Construct the formula corresponding to additon of velocities of frames in the Einundzwanzigstein kind of theory!
f) Show that the Lawrence transformations are forming a group (Albert surely did not dare to mention this to his students...) Find the canonical parametrization of this group, and express the velocity which appears in the Lawrence transformation (both forward and backward) with this canonical speed-parameter!
g) Show that if the speed difference is small (relative to c), then the formulae of re(duce)lability are approximating the transformation formulae of the Newtonian classical mechanics!
h) Show that, contrary to appearance, there is no special reference frame which is 'absolutely at rest', that is, every inertial reference frames which move at constant velocity with respect to each other are similar.
i) We have seen that the theory of Albert Einundzwanzigstein is much simpler and presentable than that of Einstein. What is the reason why Einstein did not discover the first one? What is the main difference between the two theories?
j) Mynden Lee ben Canal, the renown science critics states that the theory of Einundzwanzigstein – though it seems rather curious – is not at all new. Lee ben Canal has heard about a proof, which says that under some reasonable mathematical assumptions, there can only exist two types of transformations, which is the Galilei-type and the Lorentz type. The Lawrence transformation then, which appears in the new theory, must be isomorphic to any of the two. Is this statement true? If not, why not? If yes, with which theory can the re(duce)lability theory be identified with? Write down the formulae, with which the two theories can be transformed to each other!

(Gyula Dávid)

21. (Continuation of the previous problem). Work out the kinematics and point-mechanics of Einundzwanzigsten's re(duce)lability theory!
a) Introduce vectors and tensors with upper and lower indexes, define scalar product, metric tensor, etc.
b) (independently of part a)) Write down the action integral of the freely moving particle! Pay attention to the principle of correspondence: at small velocities, the formulae must reproduce corresponding formulae of classical mechanics.
c) Based on the action function, define the momentum and energy of the free pointlike particle! How do these quantities are transformed under the Lawrence transformation?
d) Study the correctness momentum- and energy-conservation theorem!
e) Write down the equation of motion of a re(duce)lativistic particle moving in homogeneous external gravitational field, and solve the equation!
f) Write down and solve the equation of motion of a particle moving in the (1+1) dimensional electromagnetic field

(Gyula Dávid)

22. Let us solve the eigenvalue-problem of the electromagnetic field strength tensor which appears in relativistic electrodynamics! Separate the eventually appearing singular special case(s)! What is the physical meaning of the eigenvalues and eigenvectors? How do the eigenvalues relate to the known invariant quantities of the electromagnetic field? Repeat such studies with the energy-momentum tensor of the electromagnetic field in vacuum, exploiting the previous results! (Suggested literature: L. D. Landau and E. M. Lifshitz: Mechanics and Electrodynamics, Franklin Book Company, 1972.)

(Gyula Dávid)

23. How far can an astronaut reach from Earth during his lifetime? Let us assume that such rocket technology is available which can provide constant acceleration for an arbitrarily long time! (The problems of rocket fuel and other disturbing details can be disregarded!) Let us choose reasonable parameters, such as 30 years of travel, acceleration equals to the terrestrial gravity! Also consider the recently mostly accepted Universe model with accelerating expansion (Lambda-CDM) with its presently known parameters!

(István Csabai)

24. Consider a circle shaped p(n)-type semiconductor with a radius, R. The upper layer with thickness t is doped by ions of an appropriate atom providing an n(p)-type upper layer. An important parameter of this p-n junction is the sheet resistance of the upper layer, $R_s = \rho/t$, where ρ is the resistivity of the upper layer. Spreading of carriers in the upper layer are determined by two other parameters: C_d and R_d is the capacitance and resistance of p-n junction, respectively.

Let the p-n junction be illuminated by light with an appropriate wavelength (energy of a photon should be higher than the band gap) and chopped by a frequency f to get a square wave illumination. Radius of light spot is r_L. Electron hole pairs generated by illumination are diffusing to junction and going to be separated: electrons are going to n-type layer and holes to p-type layer. Upper layer is thin and it is a very good approximation that carriers are spreading only laterally. Changing in potential by spreading carriers can be described in first approximation by a two dimensional Helmholtz-equation (parasitic capacitances to ground and air are neglected alongside the effect of spreading of carriers in the lower layer):

$$(\Delta - a^2)U(r,\phi) = -JR_s\Theta(r_L - r),$$

where J is the generated photo current, Θ is the Heaviside and

$$a = \sqrt{\frac{R_s}{R_d} + i2\pi f R_s C_d}$$

is the characteristic wave number of spreading signal. Signal is detected by two electrodes capacitively, one of them is a transparent circle shaped with the same radius as for light spot and the other one is a ring shaped with inner and outer radius, r_1 and r_2, respectively. Let assume that measuring electrodes are infinitesimally close to sample and parasitic capacitance caused by electrodes are neglected. It is assumed that carriers cannot go away from upper layer at the edge of sample, i.e. gradient of signal is zero in direction of a radius if the center of coordinate system is the center of sample.

a) Plot effective resistance $R_{\text{eff}} = \frac{|U|}{Jr_L 2\pi}$ for transparent and ring shaped electrodes as a function of frequency for $f = 500000/2^n$ and $n = 0..15$, assuming that illumination is in center of sample.

b) During mapping of sample measuring head is moving above sample, i.e. location of illumination and electrodes are changing. Let r_0 be the distance between center of sample and light spot! Determine the expression of spreading signal in r_0 based coordinate system! Plot the effective resistance as a function of r_0 in the region of $[0, R - r_L]$ along a radius for $f = 7800$ Hz.

Typical values of parameters: $R_s = 500\,\Omega$, $C_d = 5\,nF/\text{cm}^2$, $R_d = 300\,k\Omega\,\text{cm}^2$, $R = 150$ mm, $r_L = 1$ mm, $r_1 = 2.5$ mm, $r_2 = 5$ mm. Pay attention that light spot and electrodes have a finite size!

(Krisztián Kis-Szabó)

25. Let us study the movement of a particle in a lens-shaped billiard (the particle can move inside the billiard without friction, and it bounces off the walls perfectly elastically). The billiard is bounded by two arcs of the same radii, which are connected at vertices where the arcs meet. Examine the case when the length of the arc is larger than the semicircle, too. Perpendicularly to the line connecting the vertices, along the symmetry axis, a periodic orbit emerges.

a) Study the stability of the orbit!

b) What shape is formed in one section of the phase space by those orbits, which fall close to this periodic one, provided that the periodic orbit is stable?

c) Determine approximately the energy eigenvalues of the quantum states which are concentrated on the periodic orbits which appear in part b).

(Zoltán Kaufmann)

26. Let us try to find a one dimensional classical mechanical system, in which translation of time results in a scale transformation in the following form: $x \to K(t)\cdot x$. First determine the $K(t)$ function, exploiting the fact that time translations forms a group, then write down the Hamiltonian $H(x,p)$ which generates the required dynamics. After that let us quantize the system (taking care of the hermiticity of the Hamilton-operator). Determine the $\psi(x)$ energy eigenfunctions! How do the above mentioned scale transformation $\psi(x) \to \psi(\frac{x}{K(t)})$ acts on these functions? Finally let us consider the $\frac{1}{n}$-type scale changes, and prescribe the $\sum\limits_{n=1}^{\infty} \psi(nx) = 0$ quantization condition! Which will be the energy-eigenvalues? And what would Riemann, Hilbert or George Pólya say to all that?

(Gábor Kónya)

27. Determine the energy levels of an electron confined in a box with the sides of the box given as a and b, height h, and a homogeneous vertical gravitational force acting on the electron! The sides of the box can be regarded as infinitely high potential walls. Study the high energy (semi-classical) limiting case!

(József Cserti)

28. Let $\mathbb{K} = \mathbb{C}$ or $\mathbb{H}$, where $\mathbb{H}$ denotes the quaternion. Consider the 2×2 hermitian matrices above $\mathbb{K}$ of the following form:

$$A = \begin{pmatrix} t+x & y \\ y^* & t-x \end{pmatrix} \qquad \bar{A} = \begin{pmatrix} -t+x & y \\ y^* & -t-x \end{pmatrix},$$

where $x, t \in \mathbb{R}$ and $y \in \mathbb{K}$. Let us show that the linear space of matrices of these types can be naturally identified with the 3+1 or 5+1 dimensional Minkwoski-space! What is the expression which determines the $g(A, B)$ Lorentz metric?

Let us define spinors in the following way. Let be $\phi, \psi \in \mathbb{K}^2$ and to a given A hermitian matrix let us assign the $\gamma(A)$ matrix, which acts on the $(\phi, \psi) \in \mathbb{K}^4$ pair in the following way:

$$\gamma(A)(\phi, \psi) = (A\psi, \bar{A}\phi).$$

Let us show that the γ matrices defined in this way fulfill the Dirac-identities, that is,

$$\{\gamma(A), \gamma(B)\} = 2g(A, B) \cdot \mathbf{1}.$$

How can the Lorentz transformation be represented on the space of spinors? How can one produce scalar or vector quantities from two spinors, similarly to the expressions $\bar{\Psi}\Psi$ nd $\bar{\Psi}\gamma_\mu\Psi$ in the Dirac-theory? Let us show that such expressions are covariant!

Finally: how about the case of octonios?

(Balázs Pozsgay)

29. Noninteracting bosons trapped in a 3 dimensional harmonic oscillator potential can show Bose-Einstein condensation below a critical temperature T_c.

a) Show that the critical temperature is given in leading order ($T_c \approx T_0$) by $k_B T_0 = K N^{1/3}$ as a function of the particle number N. Calculate the proportionality factor K.

b) According to more precise calculations the critical temperature T_c gets finite size corrections. Give an estimate (calculate the magnitude) of $(T_c - T_0)/T_0$ as a function of N.

(András Csordás)

30. Electron dynamics in a two dimensional sample of surface A is given by the Hamiltonian operator of the following form:

$$\hat{H} = \frac{A}{(2\pi)^2} \int\limits_{\mathbf{k}\in\Omega} \mathrm{d}^2 k \Big[\epsilon(\mathbf{k}) \left|\mathbf{k}\uparrow\right\rangle \otimes \left\langle\mathbf{k}\downarrow\right| + \epsilon(\mathbf{k})^* \left|\mathbf{k}\downarrow\right\rangle \otimes \left\langle\mathbf{k}\uparrow\right| \Big],$$

where X^* denotes the complex conjugate of X, $|\mathbf{k}\uparrow\rangle$ ($|\mathbf{k}\downarrow\rangle$) denotes the $\uparrow$ ($\downarrow$) spin states of definite momentum $\hbar\mathbf{k}$, and

$$\epsilon(\mathbf{k}) = \epsilon_0 \left[1 + 2\cos(k_x a)\exp\left(i\sqrt{3}k_y a\right)\right],$$

where ϵ_0 is a constant with energy dimension, and a is another constant with length dimension ($a \ll \sqrt{A}$). The possible wave-number vectors $\mathbf{k}$ of the electron form a domain with shape of regular hexagon (Ω) with apexes are given with the following coordinates: $\mathbf{K}_n = \frac{2\pi}{3a}\left(\cos\left(\frac{\pi}{3}n\right), \sin\left(\frac{\pi}{3}n\right)\right)$ (n=0,1,...5).

a) Determine the group velocity $\mathbf{v}_g(\mathbf{k})$ of the electron!

b) We are preparing inside the sample a wave packet which is narrow both in coordinate and in momentum space:

$$\Psi(\mathbf{r}) = \frac{1}{2\sqrt{\pi}\Delta r} \begin{pmatrix} e^{-i\alpha(\mathbf{k}_0)} \\ e^{i\alpha(\mathbf{k}_0)} \end{pmatrix} e^{i\mathbf{k}_0\mathbf{r} - \frac{(\mathbf{r}-\mathbf{r}_0)^2}{4\Delta r^2}}, \qquad \alpha(\mathbf{k}) = \sqrt{\frac{\epsilon(\mathbf{k})}{|\epsilon(\mathbf{k})|}}.$$

Investigate the spatial outspreading of the wave packet in the as a function of time! Which is the $\mathbf{k}_0$ parameter at which the outspreading is the slowest?

(Péter Rakyta and László Oroszlány)

31. A few years ago, the University of Faraway was relocated to new premises.

Prof. Dr. Nosey Parker, professor of meta-para-physics was given an office at the first floor with a panoramic window looking to the north, and has been showing off his room, bathing in sunlight all the year round, to every visitor ever since. ‘It is such a joy to work here’, he says. ‘In this wonderful sunshine the complicated formulae of meta-para-physics hop from my mind to the paper as if of their own accord.’

Clearly, the professor’s joy turned to sorrow when construction begin on a stately, nine-storey building of the rival Ortophysics University just opposite his window and barely fifty meters away. Prof. Dr. Nosey Parker already foresaw the sad moment when the Sun is going to be forever hidden by grey concrete walls.

He reached a decision: ‘I cannot let this happen!’, and immediately set to work.

Before long, the space between the two buildings filled with complex mechanical, thermal and optical devices, all intended on manipulating the refraction index of the air.

‘If the refractive index and its spatial dependence is set up properly, the light rays passing above the neighboring building turn downwards and again illuminate my room all the year round. If I am suitably adept they will even reach the hind wall. Of course, there could still be some technical difficulties’, the professor explained to the interested students of the Department of Para-Optics. Sure enough, all of the students offered their help.

Help the professor yourself! Compute how to change the refraction index of the air between the buildings! What technical manipulations and devices can realize the goal? Perform numerical estimates too! What are the risks and side effects of these manipulations?

For your information: the University of Faraway is located at latitude 49° S. The temperatures vary between 42°C for the hottest summer days, and −25°C for the coldest winter days. Faraway’s office building standards stipulate that the headroom must be 4.2 meters, and university office must measure 5 meters x 5 meters.

(Gyula Dávid)

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