THE 38th—10th INTERNATIONAL— RUDOLF ORTVAY PROBLEM SOLVING CONTEST IN PHYSICS 2007

The Physics Students' Association of Eötvös University, Budapest, the Roland Eötvös Physical Society and the Hungarian Association of Physics Students proudly announce the **38th** – and for the tenth time **international** Rudolf Ortvay Problem Solving Contest in Physics, between 26 October 2007 and 5 November 2007.

Every university student from any country can participate in the Ortvay Contest. PhD students compete in a separate category. The contest is for individuals: solutions sent by groups of students are not accepted. The name, the university, the major, and the university year should be indicated on the solutions. Pseudonyms and passwords cannot be used: each contestant has to use his/her own name.

The problems can be **downloaded** from the webpages of the Ortvay Contest

http://ortvay.elte.hu/

in Hungarian and English languages, in LaTeX pdf and Postscript formats, from **12 o'clock (Central European Time, 10:00 GMT), Friday, 26 October 2007**. The problems will also be distributed by local organizers at many universities outside of Hungary.

Despite all the efforts of the organizers, it may happen that some unclear points or misprints stay in the text. Therefore it is very useful to visit the webpage of the contest from time to time, as the corrections and/or modifications will appear there.

Each contestant can send solutions for up to 10 problems. For the solution of each problem 100 points can be given.

Any kind of reference material may be consulted; textbooks and articles of journals can be cited.

Each problem should be presented on (a) separate A4, or letter-sized sheet(s). The contestants are kindly asked to use only one side of each sheet. Solutions written by pencil or written on thin copy-paper will not be accepted as these cannot be faxed to the referees.

Computer programs appended to the solutions should be accompanied by detailed descriptions (what computer language it has been written in, how to use the program, which parameters can be set, what notations are used, how to interpret the output figures and graphs, etc.) They can be enclosed on floppy disks, or sent via email to the addresses below.

Solutions can be sent by mail, fax, or email (in LaTeX, TeX, pdf or Postscript formats). Contestants are asked not to use very special LaTeX style files unless included in the sent file(s). Electronically submitted solutions should be accompanied—in a separate e-mail—by the contents and, if necessary, a description explaining how to open it.

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Deadline for sending the solutions: 12 o'clock CET (11:00 GMT), 5 November 2007.

Contestants are asked to fill in the form available on our webpage after posting their solutions. It will be used for identification of contestants and their solutions. **Without filling in the form, the organizers cannot accept the solutions! The form is available only on 5 to 6 November.**

The contest will be evaluated separately for each university year, according to the total number of points. The referees reserve the right to withhold, to multiply or to share some prizes. Beyond the money prizes given for the first, second, and third places, honorable mentions and special prizes for the outstanding solutions of individual problems can be awarded. This is why it is worthwhile sending even one or two solutions.

The announcement of the results will take place on 7 December, 2006. The detailed results will be available on the webpage of the contest thereafter. Certificates and prizes will be sent by mail.

Wishing a successful contest to all our participants,

the Organizing Committee: Gyula Dávid and József Cserti
The original Hungarian text is translated by Katalin Kulacsy, József Zsolt Bernád, András Csordás, Zéno Farkas, Zoltán Kaufmann, András Pályi, Zoltán Rácz, Dezső Varga

1. The next morning they were already standing there...

   Sunday morning the opening ceremony went smoothly, flags, dances, leading politicians from all over the world, only a few groups of terrorist, lofty speeches about the Gate of the Solar System, the True Beginning of Space Age, the Second and this time Final Gate Opening when even simple people and their home made scientific devices, or simply their cherry to be sold on the space stations could get through to space at little cost and in great mass... Finally the ribbon was cut, officially putting into operation the first Space Lift, a ribbon made of carbon nano-tubes stretching between the space station on its synchronous orbit and the equatorial station below it (Lifthof, as a journalist named it), on which passengers and goods could travel slowly, comfortably, at low speed and low load, therefore at low cost to the space transfer stations and thence to distant regions of the Solar System. The last champaign was drunk, the last piece of music was played, everybody went home and the next day the actual operation of the space lift could start.

   The next morning when the chief engineer arrived they were already standing there. Forty to fifty weird fellows, faces tanned by sun and wind, with equipment looking half professional, half makeshift, and with much resolve. They all wanted to travel by the space lift but not to the space station, as they wanted to bail out on the way... And they were willing to pay for this.

   The chief engineer listened to Joe Jumper, the leader of the group of skydivers, flabbergasted. They were the ones who threw themselves down from sky-scrapers, radio and TV towers and high cliffs into the depths, to open their parachutes after a couple of seconds or of dozens of seconds of free fall and descend to the foot of the wall. Of course only when the wind or a failed jump did not fling them at the tower or cliff earlier. Sometimes they travelled and climbed days for that little free fall. And now here was this ten thousands of kilometres high, literally sky-high tower that could offer as much as several hours of free fall – of course they felt it natural that it had been built for them to jump from it, from as high as possible. They procured or knocked together a couple of used space suits, thermal protection shields, and here they were standing, ready for the jump.

   Of course the chief engineer drove them away. Just like the following days. When he had to stop the forthcoming space transportation task for the forty-second time, however, in order to oust the stowaway skydiver from the shipment, he started to ponder seriously. Here is a huge demand (by that time several hundreds of skydivers were already waiting each morning, looking longingly at the slender thread stretching to the sky), and they can satisfy this demand with a little extra investment. It is enough to build a few platforms along the path of the lift – one for beginners, at an altitude of a couple of miles, one for advanced skydivers in the high atmosphere and one of course for professionals, at the highest possible point from which it is still possible to jump to the Earth. On the last sixty miles the heat shield and then the parachute of the skydivers will play the lead but that's their problem, insurance will be made mandatory...

   After some hesitation the governing board of the Space Lift deemed the idea useful from the PR point of view and consented to the constructions, thus thought was soon followed by deeds. Within a couple of months the platform at the maximum altitude was ready and the skydivers (space jumpers as they called themselves from then on) stepped off the platform with happy faces to fly into nothingness.

   Questions: a) How high was the platform built? b) How long did the jump last (ignore the last appr. sixty miles covered in the dense atmosphere, this depended mainly on the individual equipment and skills of the jumper)? c) How far did they land from the bottom terminus of the space lift, the Lifthof?

   There were some who kept swarming there. As soon as they landed they travelled to the Lifthof and booked the next ascension. The idea soon arose that it would be much simpler if they could land right at the bottom of the tower. They at once started to press for the construction of a new platform. The chief engineer opened his arms in regret: alas, the laws of physics... But then he quickly found the solution: instead of simply dropping down, a horizontal push off from the platform should be provided (by some technical aid and initial velocity).

   Question: d) At what speed?

   For a while jumping went on undisturbed, space jumpers got used to the launching device, the number of accidents and casualties stabilised at a constant value relatively low compared to other extreme sports. But then the restless Joe Jumper realised that the last sixty miles, the atmosphere and the landing only complicated the adventure unnecessarily. Let us rise higher, jump from higher! - was his new slogan. The others warned him that if he avoided the Earth he could stay orbiting it forever. He, however, had heard something when he was a university student that after a complete period the moving body would return to its original position. I'll catch the platform where I started, drink a cup of coffee and jump again, again without a kick in the bottom – he said beaming. The chief engineer tried again to dissuade him on the basis of the laws of physics, but, as he knew it in advance, in vain. The new platform was soon finished, higher than the old one, but at the lowest point where the requirements of the fans of this newest sport, Permanent Space Jump, could be satisfied: to return to the starting platform after the jump.

   Questions: e) How high was the new platform built? f) How long did the oxygen packed by the jumpers have to suffice? g) When and at what altitude did they get closest to the Earth? Additional questions: h) Draw the paths of all three kinds of jumpers as seen from the co-ordinate system of the geostationary space station spanning the cable of the space lift.

   (Gyula Dávid)

2. How and why do sausages rotate around their longitudinal axes in hot (but not boiling) water? Make experiments, observe and explain the phenomenon. (Suggestion: let the sausage be a simple poultry sausage in artificial casing with the casing stripped off. Put salt into the water.)

   (János Koltai)

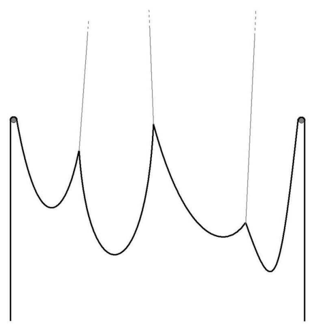
3. A torus of homogeneous material (having radii R and r, $r << R$, where r is the radius of the torus tube, R is the radius from the center of the hole to the center of the tube) is freely sliding on a horizontal plane, its initial velocity is v. The torus is also spinning around its own symmetry axis, its initial angular velocity is ω_0. The coefficient of kinetic friction is μ. The drag coefficient of the air can be neglected. How does the torus' center of mass move? Calculate its trajectory.

(Gábor Veres)

4. A sizeable spaceship is on orbit around the Earth. What is the force acting on a point-like object in the coordinate system of the spaceship? What is the trajectory of a free point-like object?

(Géza Tichy)

5. A long chain hangs on two nails drove in at identical heights, and the part of the chain between the nails is tied to the ceiling by means of very long threads of different lengths according to the figure. Make a sketch of the shape taken up by the chain in the state of static equilibrium. (The threads can slide freely along the chain, friction is negligible everywhere. The chain is sufficiently long to "hang loose" between two threads.)



(Máté Vigh)

6. Three small rail-cars can move without friction on a sufficiently long pair of rails. The ends of the rails are closed by perfectly elastic buffer stops and the cars are connected by springs with identical angular frequencies ω and lengths l. Push all three small cars at an identical speed v in the same direction and describe the motion.

   a) Describe the motion of the cars between the first and the second collisions. When does the second collision occur? (Assume that two cars can never collide, i.e. $3v^2 < \omega^2 l^2$.)
   b) Investigate numerically how many collisions can take place before the velocity of the centre of gravity changes direction and the three cars start moving towards the other buffer stop. (Write an algorithm calculating the data of a collision based on the parameters of a previously occurred one instead of simulating the time evolution.)
   c) Examine the system after a long time period. What is the average energy stored in each mode? Is the Boltzmann distribution realised? Try to give a theoretical prediction as well to this question.

(Balázs Pozsgai)

7. A linear chain is built up such that neighbouring and second-neighbouring objects are connected by springs. The lengths of springs are a between neighbouring objects and $b \neq 2a$ between second neighbours. What is the distance of the objects if the chain is not under tension?

(Géza Tichy)

8. A box of paint placed on a hillside with uniform slope m is heated up by the strong sunlight. The box suddenly blows up. The drops of the paint start independent trajectories, uniformly distributed in every directions, with equal initial speed v. What is the shape of the stain formed by the paint drops on the hillside?

(Zoltán Kaufmann)

9. On a space station the child of the commander wants to play skipping-rope. Although she cannot jump, she turns the rope at a constant angular velocity. What will the shape of the skipping-rope be?

(Péter Gnädig)

10. Is it possible to make a juggling machine? In principle, nothing else is needed but a pair of large mass plane objects moved by a machine in a pre-defined rhythm, up or down or even sideways, while nearly perfectly elastic balls bounce off them according to an appropriate pattern.

    The simplest “one-dimensional” juggling machine is made up of one horizontal plane moved upwards for a time period $T/2$ and moved downwards for a time period $T/2$, both at speed V. If we release a ball above that structure, it starts bouncing. The coefficient of restitution is κ (ratio of relative velocity after and before the collision). If appropriate parameters are chosen, stationary motion will develop. What are the conditions for that? When will such a motion be stable?

    A simple, one-ball, but two dimensional juggling machine can look as follows: take two planes at distance L, incline them at angles α and $-\alpha$, then move them in a direction perpendicular to their plane up and down at a velocity V, periodically but with a half period time delay between them (when one is up, the other one is down). Which are the conditions of a stationary trajectory, and at which parameters will this trajectory be stable? Try to show (numerically) that above a critical value α_c the system is unstable at any value of κ (the coefficient of restitution is assumed to be the same on both sides).

    Propose other arrangements! How can one comment on the constructions seen here `http://youtube.com/watch?v=sBHGzRxfeJY` ?

    (Balázs Pozsgai)

11. The Hamiltonian of a point-like particle is an arbitrary function of the momentum, that is, of the form $H = f(\mathbf{p})$. What is then the Newton’s law in the presence of electric and magnetic field?

    (Géza Tichy)

12. Estimate the change in the rotational period of the Earth due to a volcanic eruption. Can this effect be measured by means of some interferometer? What should the parameters of such an interferometer be?

    (József Cserti)

13. Windmills make use of part of the air’s kinetic energy. Investigate the theoretically achievable maximum efficiency of windmills by means of the following simplified one-dimensional model. Let the wind blow at right-angles to the surface of the windmill and let the speed of the air be V at a great distance before the windmill and αV $(0 \leq \alpha \leq 1)$ far behind it. Along the surface of magnitude A of the windmill let the speed of the wind be v.

    a) Express the ratio v/V in terms of of α.
    b) Determine the theoretically achievable maximum efficiency of the windmill in this model.

    Guidance: The efficiency of windmills is characterised by the amount of energy they can take up from a mass of air of cross-section A coming from a great distance in the direction perpendicular to their wheels.

    (Gyula Honyek)

14. Teal’c has been taken prisoner by a goauld who is keeping him on his ship. The goauld does not care about Teal’c’s situation, it only wants to occupy his brain. Teal’c has already noticed that his mind had been intruded into. He attempted to resist using the ancient jaffa technique but this was not enough. Since he does not want to yield his mind and body to the enemy, he decides to commit suicide.

    He first orientates. He is sitting in a chair in a cubic room with $a = 5$ m long sides, on the “bottom” of the room, at the centre of the square base of the cube. The temperature of the air in the room is 300 K, its pressure is 100 kPa, its composition is 20 % O_2 and 80 % N_2. Teal’c knows that on the other side of one of the walls of the room there is Space. He decides to remove the side of the room and die due to the escape of air.

    He can do this by two means: a) if the wall is a sliding door he can push it aside within 1 thousandth of a second; b) he simply detaches the wall, and as nothing joins it to the ship any more, the inner pressure starts to push it away.

    SG-1 (StarGate-1) is near at hand and could save Teal’c but this takes time. They are asking physicists how much time they have after removing the wall to save Teal’c. They know that when the air escapes neither the chair nor Teal’c, who is fastened to it, move. They also know that due to the physical damages suffered during a fast decrease of the pressure a man dies at a pressure of 20 kPa. But Teal’c is not human, he only perishes at a pressure of 10 kPa.

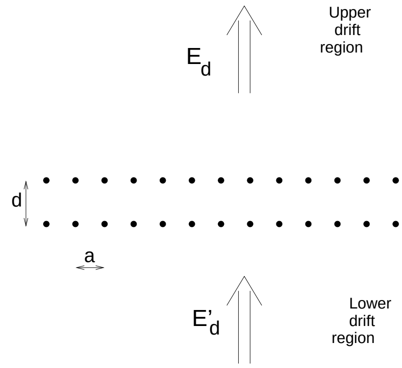
    How much time would SG-1 have to save Teal’c if this latter were human? How much time does SG-1 have in the case of Teal’c? Does this time depend on the side length of the cube? If it does, how? For what side length is this time the longest?

    (István Horváth)

15. Prove that if the solid material is formed by pair-potential among the constituents, then the components of the C_{ijkl} elastic modulus tensor do not depend on the index ordering (thus are invariant under any index permutation)!

(Géza Tichy)

16. A periodic electrode structure is prepared, according to the figure, which is translation symmetric with respect to an axis perpendicular to the plane of the paper. The negligibly (but not infinitely) thin wires (of radius r) are connected to potentials of $+U$ and $-U$. Far from the plane of the wires, large plane electrodes provide a homogeneous electric field. Let us call these homogeneous regions above and below the wires, with electric fields of E_d and E'_d, drift regions. We are interested in the case when U is rather large, typically $U/d > E_d$.



a) The electrodes are surrounded with a clean gas, within which ions, once created, drift slowly due to the electric field. Consider negative ions: the velocity of these is in every moment proportional to the electric field:

$$\mathbf{v} = -\mu \mathbf{E}$$

The constant μ (called mobility) depends on the properties of the gas, we consider it as given. We can neglect for the moment also the electric field generated by the charge of the ions itself (i.e. the space charge).

How do the trajectories of the ions starting out from the upper drift region look like? What fraction of the ions will reach the lower drift region?

b) Now consider electrons, which in clean gas behave similarly to ions, but their diffusion is not negligible (still their mean free path is short enough and negligible). This means that only their average velocity will be equal to the electric field:

$$< \mathbf{v} >= -\mu \mathbf{E}$$

and a diffuse motion will be added to their displacement, due to which in a short time t they will on average move away to a distance of $\eta\sqrt{t}$, in a random fashion. Which fraction of the electrons starting from the upper drift region will get through to the lower drift region?

c) Qualitatively, what phenomena are expected if the electric field of the ions' or electrons' own charge is also taken into account (this is called space charge)? E.g. how will the fraction of ions/electrons that get to the lower drift region change? How will the speed of transfer, or the electric field observed by the ions/electrons among the wires change?

(Dezső Varga)

17. On the basis of geometrical optics calculate the position where a point-like light source should be placed in a spherical medium of radius R and negative refractive index $n < -1$ so that light beams emerging from the sphere cross in one point. Study the problem on the basis of the Maxwell equations as well, i.e. in the case where R is comparable to the wavelength of the light.

(József Cserti)

18. A diffraction grating is built up from adjacent strips of width a. The strips, independently from each other, are either dark with a probability of p, or light with the probability of $1-p$. What is the Fraunhofer-type diffraction pattern?

(Géza Tichy)

19. A mirage appears above the flat land of Hortobágy on a warm day. As it is well known the refractive index of the air decreases upwards as a function of the height z according to

$$n(z) = n_0\sqrt{1 - \frac{z^2}{h^2}},$$

where n_0 is the refractive index at the ground and h is a parameter with dimension of length. What are the trajectories of the light rays starting from a point-like lamp on the ground? Meanwhile some light fog appears in the air. What is the shape of the region in the fog which is illuminated by the lamp?

(Zoltán Kaufmann)

20. A beam of light propagates perpendicularly through a system built up from an infinite number of parallel glass plates of thickness a and of refractive index n. All the plates are separated from each other by parallel air gaps of width b and of refractive index $n = 1$.
a) Show that there are frequency intervals for which the light cannot propagate through the system.
b) Calculate analytically the upper and lower boundaries of the allowed and the forbidden frequency intervals for the following cases: i) $n = 2$ and $b = a$, ii) $n = 2$ and $b = 2a$.

(Gyula Dávid)

21. A general triangle-shaped frame is constructed from thin conducting bars. A constant current flows around in the bars. Where can one find the smallest value of the magnetic induction within the triangle (in its plane)?

(Máté Vigh)

22. Consider a Michelson–Morley experiment such that all the light source (S), the semi-silvered mirror (M_0), the two mirrors (M_1 and M_2), and the light detector (D) are placed on separate rockets of exactly identical mass and power. In the first situation all the rockets are at rest relative to a given inertial frame of reference; and we observe a given interference pattern in the detector. (Fig. 1). Then, we turn on the five engines for a while, simultaneously, and turn them off simultaneously. Consequently, the rockets are accelerated to the same velocity v (Fig. 2). Do we observe any change in the interference pattern?

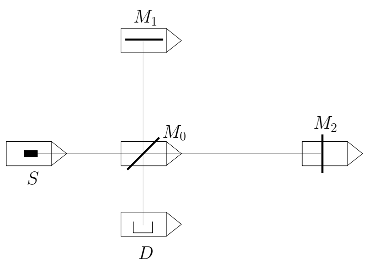


Figure 1

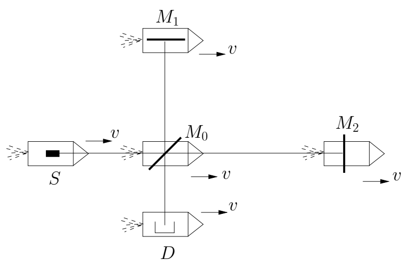


Figure 2

(László Szabó)

23. We know that two independent Lorentz-invariant scalars can be formed from the electromagnetic field strength tensor $F_{kl} = \partial_k A_l - \partial_l A_k$:

$$P = -\frac{1}{4}\,F_{kl}F^{kl} = \frac{1}{2}\left(\mathbf{E}^2 - \mathbf{B}^2\right), \quad \text{and} \quad Q = -\frac{1}{4}\,F_{kl}\bar{F}^{kl} = \mathbf{EB},$$

where $\bar{F}^{kl} = \frac{1}{2}\,\varepsilon^{klpq}F_{pq}$ a is the Hodge dual of the field strength tensor. Let the Lagrangian of electrodynamics in vacuum be the linear combination of the two invariants:

$$\mathcal{L} = \alpha P + \beta Q - \frac{1}{c}A_k j^k \ \ !$$

Formulate the field equations and compare them to the usual form. What will be the linearly and the circularly polarised plane wave solutions of the equations? Investigate the case $\alpha = 0$, $\beta = 1$. Repeat the calculations in 2+1 dimensions (two spatial and one time dimension) as well. What symmetries does the modified action integral have, and what others does it not?

(Gyula Dávid)

24. Let's assume that the curvature of the space of the Universe has a constant positive value everywhere, so that the known physical world is actually the "surface of a four-dimensional sphere", and we can observe only a part of it, which can be considered approximately planar. Are the laws of electrostatics, as we know them, compatible with this model of the Universe? Analyse the electrostatic field of a point charge in the whole "surface of the sphere". Determine the criteria of the existence of possible configurations of charges! Describe a general procedure for calculating the electrostatic field of a configuration of charges. Using these results, describe the field of a single electric dipole. First answer these questions considering a two-dimensional Universe, being the surface of a three-dimensional sphere. Is there a similar procedure to extend the equations of magnetostatics to the whole space of the curved Universe? Bonus question: The Faraday cage shields its inside only. How is this compatible with the inside-outside symmetry of the "surface of the sphere"? (In other words, how does the Faraday cage know which part is its inside?)

(Péter Kómár)

25. The following model has been proposed as a possible non-linear generalisation of relativistic electrodynamics: Let E_0 be a natural constant with the dimension of field strength (the so-called maximum field strength). Let the Lagrangian of electrodynamics in vacuum be of the form

$$\mathcal{L} = E_0^2\sqrt{1-\frac{F_{kl}F^{kl}}{2E_0^2}} - \frac{1}{c}A_k j^k.$$

Formulate the "material" equations and deduce the equivalent of the Maxwell equations with sources in 3 D form. What will the form of Coulomb's law be? And the dispersion relation of plane waves? What experiment could discriminate between phenomena described by this non-linear electrodynamics and the usual Maxwellian theory?

(Gyula Dávid)

26. Estimate the possible error of the mass measurement due to the uncertainty relation using a classical beam and scales! How does this measurement error increase with temperature?

(Géza Tichy)

27. Electron plane wave propagates in an infinitely thin quantum wire. The Hamiltonian describing the motion is $H_0 = p_x^2/2m$. The electron in the wire interacts with a finite-range static spin-dependent potential V. As a result, the electron wave is partially transmitted and partially reflected. Assuming that the incoming wave is in a pure spin state with spin polarization vector $\mathbf{P}$, and that V has time reversal symmetry, calculate the spin polarization vector of the reflected wave.

(András Pályi)

28. A model for phase diffusion of a simple harmonic oscillator is provided by the master equation:

$$\frac{d\rho}{dt} = -\Gamma[a^\dagger a, [a^\dagger a, \rho]].$$

Calculate the Fokker-Planck equation of the density matrix ρ in P-representation. Determine the Langevin-equations from the Fokker-Planck equation.

(József Zsolt Bernád)

29. For given n_r, l, s numbers the fermionic creation operators $a^+_{m,\nu}$ create a particle in the (n_r, l, s) shell, where n_r is the radial quantum number, l is angular momentum (an integer number), s is the half-integer spin quantum number of the shell, $m = -l, \dots, l$ and $\nu = -s, \dots, s$. (In case of electrons: $s = 1/2$). Let us suppose that the vectors

$$a^+_{m_1,\nu_1} \dots a^+_{m_k,\nu_k}|0\rangle$$

form a basis in the Hilbert-space, where $k = 0, 1, \dots, (2l+1)(2s+1)$; $m_i = -l, \dots, l$; $\nu_i = -s, \dots, s$ and $i = 1, \dots, k$. Give expressions for the angular momentum operators L_z, and $\mathbf{L}^2$ and for the spin operators S_z and $\mathbf{S}^2$ in terms of the above introduced creation and annihilation operators. As usual, let us denote the eigenvalues of $\mathbf{L}^2$ by $\hbar^2 L(L+1)$ and the eigenvalues of $\mathbf{S}^2$ by $\hbar^2 S(S+1)$. Let us consider all the two-particle states, which are eigenstates of $\mathbf{L}^2, L_z, \mathbf{S}^2, S_z$. Give the possible (L,S) quantum number sets for these two-particle states with the degrees of degeneracy for the $l = 2, s = 3/2$ shell. Give a closed expression for the two-particle state having $L = 0$ and $S = 0$.

(András Csordás)

30. Study the conformal invariance of classical massless field theories.

a) Consider a scalar field $\Phi(x)$ in an Eucledian space-time of dimension D with a free Lagrangian $L = \frac{1}{2}\partial_i\Phi\partial_i\Phi$. Then the field equation is the D dimensional equation of Laplace: $\partial_{ii}\Phi = 0$. Show that beyond the usual symmetries of space-time there is an additional symmetry of the theory, the inversion to the unit sphere. That is, denoting the inverted image of vector x_i by $\bar{x}_i = \frac{x_i}{|x|^2}$, show that one can choose a suitable parameter κ for which the transformed field

$$\Phi'(x) = \frac{1}{r^\kappa}\Phi(\bar{x})$$

satisfies the equation of Laplace if and only if the original $\Phi(x)$ does. What is the relation between D and κ? How does it relate to the mass dimension of the field $\Phi(x)$? Which field is formed from the constant field $\Phi(x) = A$ by the inversion?

b) How should one modify the formulas to describe the inversion of a massless vector field $A_i(x)$? How do we take into account the fact that the inversion includes a position dependent local space reflection?

c) How do we describe the inversion for a massless Dirac field in four dimensions ($D = 4$)? Let the Lagrangian be $L = i\bar{\Psi}\gamma_i\partial_i\Psi$, where $\{\gamma_i, \gamma_j\} = 2\delta_{ij}$. Show that no simple scaling method is sufficient. We have to find instead an expression of the form

$$\Psi'(x) = \frac{1}{r^\delta}P(x)\Psi(\bar{x}),$$

where $P(x)$ is an x dependent operator built up from the Dirac matrices which satisfies $P^2 = 1$. What is the exponent δ and how does it relate to the mass dimension of the Dirac field?

d) Perform the following sequence of transformation steps of the space-time and of the fields defined on that. First an inversion, then an infinitesimal translation, finally again an inversion. Then any position vector x gets back close to its original place. Give the infinitesimal transformations acting on the fields. Which are the terms common for all type of fields, and which are those depending on the Lorentz transformational properties of the different fields? What is the commutator of two such "special" conformal transformations?

(Balázs Pozsgai)

31. According to certain hypotheses the shroud of Turin was Jesus'mortuary cloth, it is therefore 1970 years old. However, the age determination by means of the radioisotope ^{14}C found it to be 660 years old only, i.e. much younger, which indicates that the shroud may be a forgery. Still, it has been suggested that the fibres of the shroud have recently been attacked by bacteria that mixed fresh, undecayed ^{14}C from the air into the fibres of the shroud. Estimate the proportion of the ^{14}C atoms in the fibres used for the age determination that should originate from this modern age pollution so that it explains the deviation of the measured age from the expected one.
Guidance: Assume that the proportion of ^{14}C in the carbon content of the air and of living organisms is the same and constant. The half-life of the ^{14}C isotope is 5730 years.

(László Takács)

32. The following bet is offered at a Bookmaker for 10 \$. You are given a closed envelope and told it contains a slip with an integer N_{env} written on it. You draw 1 million times from a normal distribution with probability density given by $P(x) = \exp(-x^2/2)/\sqrt{2\pi}$ (computer with the appropriate program is provided). The largest

value you obtained x_{max} is rounded to the nearest integer N_{max} =Round(x_{max}). Next, the envelope is opened and if it is found that $N_{env} = N_{max}$ then your money is lost otherwise you get back your money and in addition you receive 100 \$.

Questions: a) What is the value of the integer in the envelope? b) Would you bet with the above conditions?

(Katalin Ozogány and Zoltán Rácz)

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