THE 32nd
– 4th INTERNATIONAL –
RUDOLF ORTVAY
PROBLEM SOLVING CONTEST IN PHYSICS
2001

The Physics Students' Association of Eötvös University, Budapest, the Roland Eötvös Physical Society and the Hungarian Association of Physics Students proudly announce the **32nd** – and for the fourth time **international** –

Rudolf Ortvay Problem Solving Contest in Physics,
from 31 October 2001, through 12 November 2001.

Every university student from any country can participate in the Ortvay Contest. PhD students compete in a separate category. The contest is for individuals: solutions sent by groups of students are not accepted. The name, the university, the major, and the university year should be written on the solutions. Pseudonyms and passwords cannot be used: each contestant has to use his/her own name.

The problems can be downloaded from the webpages of the Ortvay Contest

http://ortvay.elte.hu/
http://www.saas.hu/ortvay and
http://www.nscl.msu.edu/~horvath/ortvay.html

in Hungarian and English languages, in html, LaTeX and Postscript formats, from 12 o'clock (Central European Time, 11:00 GMT), Wednesday, 31 October 2001. The problems will also be distributed by local organizers at many universities outside of Hungary.

Despite all the efforts of the organizers, it may happen that some unclear points or misprintz stay in the text. Therefore it is very useful to visit the webpage of the contest from time to time, as the corrections and/or modifications will appear there.

Each contestant can send solutions for 10 problems. For the solution of each problem 100 points can be given.

Each problem should be presented on (a) separate A4, or letter-sized sheet(s). The contestants are kindly asked to use only one side of each sheet. Solutions written by pencil or written on thin copy-paper will not be accepted as these cannot be faxed to the referees.

Any kind of reference material may be consulted; textbooks and articles of journals can be cited. Computer programs appended to the solutions should be accompanied by detailed descriptions (what computer language it was written in, how to use the program, which parameters can be set, what notations are used, how to interpret the output figures and graphs, etc.) They can be enclosed on floppy disks, or sent via email to the address given below.

Solutions can be sent by mail, fax, or email (in LaTeX, TeX or Postscript formats – or, if it contains no formulae, in normal electronic mail). Do not use very special LaTeX style files unless you include them in your file.

Postal Address:
Fizikus Diákkör, Dávid Gyula,
ELTE TTK Atomfizika Tanszék,
H-1117 Budapest, Pázmány Péter sétány 1/A
Fax: Dávid Gyula, 36-1-3722775 *or* Cserti József, 36-1-3722866
E-mail: dgy@ludens.elte.hu *or* ortvay@saas.city.tvnet.hu

Deadline for sending the solutions: 12 o'clock CET (11:00 GMT), 12 November 2001.

Contestants are asked to fill the form available on our webpage after posting their solutions. It will be used for identification of contestants and their solutions. **Without filling the form the organizers cannot accept the solutions!**

The contest will be evaluated separately for each university year, according to the total number of points. The referees reserve the right to withhold, to multiply or to share some prizes. Beyond the money prizes given for the first, second, and third places, honourable mentions and special prizes for the outstanding solutions of individual problems can be awarded. This is why it is worthwhile sending even one or two solutions.

The announcement of the results will take place on 6 December. The detailed results will be available on the webpage of the contest thereafter. Certificates and money prizes will be sent by mail.

We plan to publish the assigned problems and their solutions in English language – to which the contribution of the most successful participants is kindly asked. The volume is planned to be distributed all over the world with the help of the International Association of Physics Students, as well as the contestant themselves. We hope this will help in making the contest even more international.

Wishing a successful contest to all our participants,

the Organizing Committee:

Gyula Dávid, Attila Piróth, József Cserti
(Eötvös University, Budapest, Hungary)

1. You are in the middle of a huge field, and being bored, you decide to play with your watch. Therefore you put bent arm in front of your nose, and
a) Each second you turn in the direction of the second hand and make a step forward. (Your arm is kept fixed, ie it turns along with you.) Where will you end up in a minute's time? Describe the described trajectory.
b) You decide to make a step (in the direction of the second hand) only for every nth tick. You do not necessarily start off at the minute sharp. How does the trajectory depend on n and the initial moment? Classify the possible trajectories.
c) What if the second hand is accelerated uniformly, ie if it makes an angle

$$\phi = \frac{2\pi}{60}\left(at^2 + bt\right),$$

with the initial direction, where $t = 1, 2, \ldots$ is time measured in seconds, and a and b are integers. Do periodic orbits exist?
d) The second hand is not accelerated any more, but it moves continually, making n rounds per minute (n is an integer), and you also move continually (in the direction of the second hand). Describe the resulting motion.

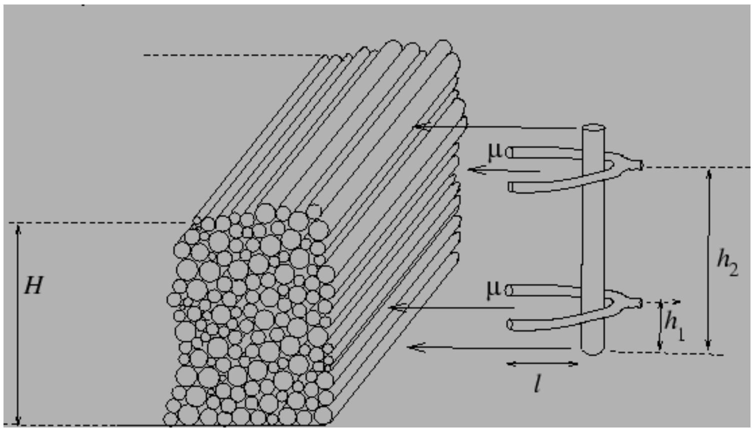
(Győző Egri)

2. A homogeneous circular disk is lying on a wall-to-wall carpet, exerting a uniform pressure on it. By pulling on the string attached to a circumferential point and pointing in the radial direction, a force F_0 is needed to move the disk. What force F can move the disk, if the string is pulled in the tangential direction? (The string is horizontal in both cases.)

(Péter Gnädig)

3. When a forest is lumbered, high and long stacks are built from the timbers previously chopped into 1-meter-long saw-logs. At the two ends of the stack large and strong trees support the logs. When there are no trees around, a simple and witty solution is used for preventing the logs from rolling in all directions.

A thick (arm-strong) bough of Y shape is placed horizontally among the logs in such a way that its two branches are inside of the stack, and in a horizontal plane, while the branching point is just outside.



Another strong bough is put through the branching vertically (see Fig.) and fixed temporarily; then the two branches of the Y are loaded in several layers with further logs. The vertical bough passes through two Ys, which, due to weight and friction, get stuck in the stack.

At what height should the two Y-shaped supports be placed? How high a stack can be built, if the branches of the Ys reach a depth l parallelly into the stack? Friction among the logs can be neglected; between the Ys and the logs the coefficient of friction is μ.

(Péter Gnädig)

4. An aircraft is gliding calmly in the night sky, when suddenly all electronics pack up (Y2K bug?), except for the steering equipment and the driving gear. The pilot cannot see, let alone hear a thing. She can rely solely on her senses, most of all her common sense, plus her middle ear (which is sensitive for accelerations). She tries to maneuver in such a manner that she should always feel a "vertical" acceleration of 1 g.

a) At what speed will the plane hit the ground if initially it was traveling horizontally at 800 km/h, at an altitude of 4000 m, but was deflected due to some slight perturbation?

b) Provided the above conditions, what trajectories can the plane follow? (Focus on planar motions and motions along the lateral surface of a circular cylinder.)

(Gyula Szokoly and Péter Gnädig)

5. A heavy rod is fixed at the floor of a stationary train through an ideal (frictionless) pivot that allows it to move in a vertical plane parallel to the direction of motion of the train. The Ant is standing on the floor next to the pivot. The Ant can exert a tiny force ε, ie it is arbitrarily feeble. Initially, the rod is in its vertical, unstable equilibrium position. Suddenly the Evil Engineer of the train (who has devilish engine-ears), whose aim is to make the rod fall on the floor, announces that the train is about to leave. It shall move along a straight path and accelerate according to the function $a(t)$ – which he even shows to the Ant. The Evil Engineer chooses the finite duration T of the experiment but the Ant decides when it should start. Can the Ant prevent the rod from falling on the floor until the experiment is finished? On which properties of the acceleration function $a(t)$ does it depend?

(Péter Fige and Ferenc Wágner)

6. A point-like particle is moving in a central force field $V(r)$. Following its motion for a while one realizes with a bit of a shock that it moves along a circular arc of radius ρ that is eccentric, ie its center is at a distance $R > \rho$ from the center of the force field.
Determine the potential function $V(r)$. What is the complete trajectory of the particle?

(Gyula Dávid)

7. Most people love to hike – but the pleasure they take in it differs from one person to the other. Suppose a hiker wants to get from point A to point B in a certain amount of time. The relief map of the area is at his disposal, ie he knows the function $h(\mathbf{r}) = h(x, y)$. (The vectorial quantities should be regarded as two-dimensional, ie z components are suppressed.) Our hiker would like to suck as little as possible, where the "suck" function is the time integral of the "labor" function:

$$S = \int_A^B L\, dt.$$

Several different labor functions can be defined. Nobody likes to run around quickly, therefore the first term is proportional to the square of the velocity:

$$L_1 = \frac{a}{2}\, v^2.$$

Going uphill sucks a lot, while going downhill is cool:

$$L_2 = b\, \mathbf{vg},$$

where $\mathbf{v}$ is the hiker's velocity vector, and $\mathbf{g} = \nabla h$ the gradient function of the terrain. When, however, the hillside is very steep, going downhill is just as difficult as going uphill, therefore:

$$L_3 = \frac{c}{2}\, (\mathbf{vg})^2.$$

(In the above expressions a, b and c are positive constants.)

a) Write up and interpret the hiker's Euler-Lagrange equations for each term above. What are the resulting paths and hiking strategies? Is the "energy" a constant of motion? If so, what is the meaning of "energy conservation"?

b) Consider a balanced hiker, whose labor function is the sum of the three above terms,

$$L = L_1 + L_2 + L_3.$$

He has to get to the around a cone-shaped mountain, ie let $h(x, y) = -G\, \sqrt{x^2 + y^2}$, where G is a positive constant. Suppose that $h_A = h_B$. What path should he follow? How should he walk along it?

c) Propose a labor function of your own, with at least one essentially new term. Sketch the implied strategies.

(Zsolt Bihary)

8. Two identical pylons are separated by 50 meters. Between the two tops a steel wire has been spanned. The maximum slack of the wire is 1 meter. How large will be the maximum slack, when a dove weighing 2 meters of wire settles down at the middle of the cable?

(Péter Gnädig)

9. A long iron rod is stretched longitudinally, fixed, and then encapsulated in a cylindrical ring of concrete coaxial with the rod and of length L. After the concrete has hardened, the rod is stopped to be pulled at its ends. Determine the strain and the stress in the assembly. Assume that the concrete and the iron stick to each other perfectly.

(András Bodor)

10. A balloon is long and of cylindrical shape when loose. What shape does it take while it is being blown up? Perform some experiments and explain the findings.

(András Bodor)

11. Water pipes used to be made of iron but copper pipes have gained popularity recently. Copper pipes usually have much smaller diameters than iron ones. How come smaller copper pipes can do the job?

(Péter Pollner)

12. I looked into the parabolic mirror of my flashlight and was rather surprised at seeing two images of myself. What happened? The flashlight was turned off, and the mirror is approximately 4 centimeters in diameter and 3 centimeters in depth.

(András Bodor)

13. According to Thomson's model of the atom, the point-like electrons move in a uniformly distributed positive charge cloud. The assembly as a whole is electrically neutral.

    Using Thomson's model, determine the equilibrium electron configurations for atoms of various atomic numbers. (Eg, for beryllium of atomic number 4, electrons in the vertices of a tetrahedron, the vertices of a square, the vertices plus the centre of a triangle, and four collinear points constitute different equilibrium configurations.)

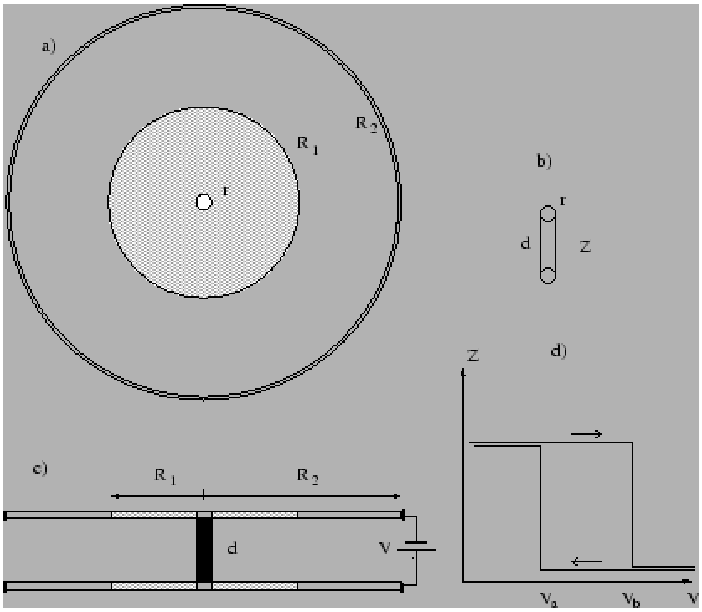
    Examine the stability of the configurations! What would the periodic table be like? Would magic numbers, halogenic elements or alkali metals exist?

(András Bodor)

14. One constructs an electromagnet from some superconducting material. The device consists of cables of finite thickness, arranged arbitrarily. The utilized "superconducting" material has the property that magnetic field can penetrate into its interior (as if it were a normal conductor), however superconduction breaks down if anywhere within the interior the magnetic field exceeds a critical value B_0. This is to be avoided. What maximal magnetic field strength can be realized outside the wires?

(Dezső Varga)

15. How to combine a bimetallic capacitor with a hystheretic resistor? The device depicted in Figs. a) b) and c) – and, for that matter, used extensively for detecting particles – consists of two identical circular condenser plates, which are parallel and separated by a distance d.



    For the radii drawn in the figure $r \ll R_1 \ll R_2$ holds. Poorly conducting material is used for the ring between R_1 and R_2, while the middle ring (R_1) and the thin rim surrounding R_2 conduct excellently. (The latter one is used for the inflow of the current.)

    Long before the experiment is started, one applies a constant voltage V to the outer rim, and waits until a stationary state is reached. Then – at $t = 0$ – one "short-circuits" the middle of the capacitor (see Fig. c)) with a cylindrical resistor of radius r, whose impedance Z exhibits hysteresis, according to Fig. d): As the voltage across the resistor is increased from zero to V_b, the initially rather large resitance drops to a negligible level; when, however, the voltage is decreased from an value higher than V_b, the impedance returns to its high (initial) value only at a lower voltage value $V_a < V_b$. Throughout the experiment the voltage applied at the thin, conducting rim is kept fixed at $V > V_b$. Determine the current through the resistance as a function of time. How does the voltage vary at radius R_1, ie the interface between the excellent and the poor conductor?

(György Vesztergombi)

16. It is well known in classical (nonrelativistic) electrodynamics that an electron traces out a circular orbit in a uniform magnetic field. But what if it is not all alone? Examine two electrons in a homogeneous magnetic field moving in the plane perpendicular to the field lines. Under suitably chosen initial conditions the two electrons are found to move along the same circular orbit. Determine the radius of this circle. Are there further periodic solutions?

    What happens in the case of an electron and a positron? Can the two annihilate (ie encounter), and if so, under what initial conditions?

    Try to give a (qualitative) description of many-electron systems, supposing again that all the electrons move in a common plane that is perpendicular to the field lines.

(János Asbóth)

17. A thin conducting ring of radius a and resistance R is dropped from height h onto a small magnet of magnetization $\mathbf{m}$ resting on a table. Let the direction of the vector $\mathbf{m}$ be perpendicular to the table top, and suppose that the axis of the ring stays vertical and points in the direction of the magnet throughout its motion. Describe the motion of the ring. When does it hit the table?

(József Cserti)

18. Plane parallel plates of refractive index n_1 and thickness d_1, and of refractive index n_2 and thickness d_2 are superposed alternately, and thus a periodic assembly is obtained. Write up an the equation determining the dispersion relation for light propagation normal to the planes. Show that optical gaps are formed, ie forbidden frequency regions, in which light cannot propagate in the periodic system of plane parallel plates. What is the quantum analog of this optical setup?

    Let $d_1 = \lambda_1/4$ and $d_2 = \lambda_2/4$, where λ_1 and λ_2 are the respective wavelengths of light of angular frequency $\omega_0 = c\,\pi/(d_1 + d_2)$ in media of refractive index n_1 and n_2. Investigate the long wavelength limit, and calculate the width of the optical gap.

(József Cserti)

19. The URL address

    http://www.padrak.com/ine/SHEEHAN.html

    contains an article putting forward some thermodynamical paradoxes and perpetual motion machines of the second kind. The paradoxes are all concerned with the second law of thermodynamics.

    Read the article and choose one of the paradoxes discussed. Find arguments supporting or challenging the author's statements.

(Katalin Martinás)

20. Billy Beery wants to keep cool the liquid nitrogen needed for his experiments, as well as his beer, in the most economical way. He is trying to devise a device that renders non-isothermal processes unfeasible.

    "I'll take a tricky shaped funnel closed at its bottom" he thinks aloud. "Then, with a blob of mercury, I trap some gas in the funnel – thus I separate it from the outside. The position of the mercury in the funnel determines not only the volume of the gas inside, but also the thickness of the mercury blob, which, in turn, determines the gas pressure. All that is left is to find a suitably shaped funnel in which the product pV is kept constant. And then, come hell or high water, the gas temperature simply cannot change."

    Help Billy to find the suitable shape of the funnel.

(Márton Kormos)

21. Let us try to describe gravitational phenomena within the framework of special relativity. (Such theories were indeed put forward between 1905 and 1915.)

    Let the gravitational acceleration be the gradient of a scalar (four-) potential. We demand that the model account for the fundamental characteristic of gravitation that force is proportional to mass, ie bodies of different mass experience the same acceleration.

    What notion of mass should be used? What unusual hypotheses on mass need be postulated? Write up the equation of motion and solve it for some simple cases (free fall and projectile motion in uniform gravitational field, free fall radially onto the Earth from outer space, etc.). Investigate the question of the perihelion advance of Mercury, and compare the results to those known from general relativity. Set up the variational principle from which the equation of motion can be derived. Examine the motion energetically as well.

    Why is this relativistic theory of gravitation unacceptable? Why do we have to deal with another (general relativity)?

(Gyula Dávid)

22. It is a fairly common lore that general relativity differs from Newton's theory of gravitation in the sole respect that instead of Newton's scalar gravitational potential it makes use of the quantities g_{kl} forming a 4-tensor. The correct incorporation of the relativity principle is, of course, a further difference – but this is also contained in the special theory of relativity.

    Motion in a tensorial field can therefore be investigated within the framework of special relativity. Construct the action integral of a particle moving in a symmetrical 4-tensor field. Derive the equations of motion. Compare the equations thus obtained with the general relativistic geodesic equations of motion of a free particle. What are the similitudes? What are the main differences?

    (Gyula Dávid)

23. Prof. Albert Zweistein's new theory, (special) relativistic hypergravitation breaks with the ancient and rather outmoded superstition which states that the gravitational force acting on a body is proportional to its mass. Hypergravity proposes that force should be proportional to the Nth power of the mass, where N can be positive, negative, or zero, integer or non-integer (further investigation into the case of complex Ns is underway.)

    The 4-force acting on a body of mass M is therefore

    $$F_k = M^N \partial_k \Phi,$$

    where the 4-scalar-like Φ is called the hypergravitational potential, and ∂_k is the 4-gradient operator.

    Write up the equation of motion of a point-like particle of mass M in a hypergravitational field. Derive a three-dimensional equation of the form *acceleration vector = something / mass.*

    Examine one-dimensional motion in a static hypergravitational field (ie, when one can find an inertial reference frame in which the field is time-independent.) Construct the hypergravitational–relativistic counterpart of the energy integral. Calculate the particle velocity for given initial conditions and hyperpotential configuration. How much time would it take to reach the Andromeda galaxy?

    (Gyula Dávid)

24. Two infinitesimal masses, separated by a distance l are placed into a homogeneous, isotropic universe of critical mass (ie, Robertson-Walker metric, $\Omega = 1$, $\Lambda = 0$). The two tiny bodies are initially "at rest" (ie, their relative speed is zero in real space – not to be confused with rest in the co-moving frame of reference). The two bodies do not change the metric (thus they do not attract each other); they do, however react to it. How far apart will they get by the time the universe has expanded to twice its initial size (ie, the length parameter got doubled)? The universe is supposed to be matter dominated.

    (Gyula Szokoly)

25. Two identical spin-half particles of mass m and gyromagnetic ratio γ ($\vec{\mu} = \gamma\hbar\vec{S}$) are connected by a weightless, rigid rod of length L. The rod is fixed at its center in such a way that it can freely rotate around an axis perpendicular to the rod. The particles interact via their dipole moments. Find the Hamiltonian operator of the system, and then determine all the eigenstates and eigenenergies. Make an exact calculation. What is the ground state? How does angular momentum conservation manifest itself in the eigenstates?

    Illustrate the spatial distribution of the density of the magnetic moment (both the magnetic moment density of either particle and the total magnetic moment density) in the representative eigenstates.
    Hint: Choose the z-axis to be parallel to the rotational axis as well as the direction of spin quantization. Do not forget about the Pauli exclusion principle.

    (Titusz Fehér)

26. A charged spinless particle moves through a uniform static magnetic field. Its motion is confined to half of the space, ie the particle's potential energy is zero within this half-space and infinite along the plane bordering it. Let the magnetic field direction be parallel to the boundary plane. Determine the particle energy spectrum. What quantities are conserved? What are the quantum numbers? How are the Landau levels modified by the spatial confinement of the particle? To what classical motions do the different quantum states correspond?
    Hint: Choose a suitable gauge.

    (József Cserti)

27. Prove theoretically or numerically that
    a) the H^- ion is stable,
    b) the H^{--} ion is unstable.
    Hunt up experimental results supporting your calculations.

    (Zsolt Bihary)

28. Examine the themodynamics of planar rotators at low and high temperatures. Determine the temperature dependence of entropy, specific heat and susceptibility. The latter can be calculated from the response to a weak magnetic field described by the interaction Hamiltonian $\hat{H}_i = h\cos\phi$. Where is the separation between low and high-temperature behavior?

    (Szabolcs Borsányi)

29. A harmonic oscillator is perturbed by the operator $\hat{H}_1 = \lambda \exp\left(-\hat{p}^2\sigma^2/2\right)$, where λ and σ are constant. Calculate in closed form the first-order correction in λ of the ground state and the first excited state.

    (Szabolcs Borsányi)

30. Consider a quantum oscillator that can be characterized by a time-dependent frequency ω,

$$\hat{H} = \frac{1}{2}\hat{p}^2 + \frac{1}{2}\omega^2\hat{q}^2 \qquad \omega^2 = \omega^2(t).$$

In each moment of time, decompose the Heisenberg picture operators into linear combinations of shift operators. Write the operators $\hat{a}(t)$ and $\hat{a}^+(t)$ – defined at arbitrary times t – as linear combinations of $\hat{a}(0)$, $\hat{a}^+(0)$, the shift operators at $t = 0$ (Bogoliubov transformation)

$$\hat{a}(t) = \mathcal{F}_+(t)\hat{a}(0) + \mathcal{F}_-(t)\hat{a}^+(0).$$

What is the probability that at time t the state defined as $\hat{a}^+(0)|0\rangle$ occurs? Study the specific case $\omega^2(t) = u + v\sin\Omega t$ numerically or analytically.
Hint: Determine the solutions of the form

$$\hat{q}(t) = \frac{1}{\sqrt{2\omega(t=0)}}\left[\hat{a}(0)f_+(t) + \hat{a}^+(0)f_-(t)\right]$$
$$\hat{p}(t) = \frac{1}{\sqrt{2\omega(t=0)}}\left[\hat{a}(0)\dot{f}_+(t) + \hat{a}^+(0)\dot{f}_-(t)\right]$$

satisfying the Heisenberg equation governing the time evolution of the oscillator. Formulate the initial conditions imposed on the coefficients for the case of an oscillator starting off from its ground state. Check whether the commutation relations are left unchanged by the time evolution. Determine the sought-for probability by using $f_\pm$ and $\dot{f}_\pm$.

(András Patkós and Szabolcs Borsányi)

31. A crystal whose lattice constant depends strongly on the particles' zero-point vibrations is termed a quantum crystal. Study the problem of quantum crystals with the aid of a one-dimensional toy model. Consider a long, periodic chain made up of particles of the same mass. Suppose that the crystal wave function is the product of one-particle wave functions centered at the lattice sites, ie

$$\Psi(..., x_j, ...) = \prod_j \phi(x_j - ja),$$

where x_j is the coordinate of the jth particle and a is the lattice constant. Suppose, furthermore, that the potential energy is the sum of nearest-neighbor pair interactions, ie

$$U(..., x_j, ...) = \sum_j V(x_{j+1} - x_j).$$

a) In terms of V and ϕ express the expectation value of the kinetic, potential and total energies per particle. Assume that the (unnormalized) one-particle wave functions are Gaussian, of standard deviation σ,

$$\phi(x) = e^{-x^2/2\sigma^2},$$

and that the pair potential is of the Morse form

$$V(x) = D(e^{-2\alpha(x-x_0)} - 2e^{-\alpha(x-x_0)}).$$

The parameters of the Morse pair potential for different noble gases are contained in the following table.

noble gas	D (10^{-21} J)	x_0 (nm)	α (1/nm)
He	0.14	0.29	21
Ne	0.50	0.31	18
Ar	1.67	0.38	15
Kr	2.3	0.41	14
Xe	3.2	0.45	13

b) Make a sketch about the Morse potential. What are the physical meanings of the parameters D, x_0 and α? What would be the lattice constant in the case of a classical lattice (ie, when zero-point vibrations could be neglected?)
c) Express the expectation value of the total energy per particle, and minimize it with respect to the variational parameters a and σ. Is the crystal stable for every noble gas? How does the crystal lattice constant and the particles' zero-point delocalization depend on the parameters of the potential and on the mass of the particles? Find a dimensionless parameter that may characterize the degree of the "quantumness" of the crystal.

Analyze the trends obtained for noble gas crystals. Hunt up the published experimental findings in favor of the calculations.

(Zsolt Bihary)

32. Within the framework of nonrelativistic quantum mechanics, one wishes to superpose two wavefunctions representing plane waves of different masses:

$$\Psi = \left[\varphi_1(m_1, \mathbf{r}, t) + \varphi_2(m_2, \mathbf{r}, t)\right] / \sqrt{2},$$

A particle mass measurement would thus give m_1 with a 50 % probability, and m_2 with another 50 %.

Perform a Galilean boost of velocity parameter $\mathbf{v}$, then a displacement operation of parameter $\mathbf{a}$, then another Galilean boost of $-\mathbf{v}$ and another displacement to get back to the original reference frame. Show that φ_1 and φ_2 obtains different phases as a result of the four transformations, therefore Ψ is also changed in a physically measurable way.

What phase difference is created between φ_1 and φ_2 by the continuous transformation

$$\mathbf{r}' = \mathbf{r} + \boldsymbol{\xi}(t),$$

and $\boldsymbol{\xi}(t) = 0$ if $t < 0$ or $t > T$?

Show that the resulting phase difference is a relativistic effect of the zeroth order in c, which does not vanish in the nonrelativistic limit.

(Győző Egri)

33. Watch the spectacular ball game played with gluons! Depict a high-energy proton–proton collision (at $\sqrt{s} = 16$ GeV) in the CM frame as protons jumping into resonances of higher mass while exchanging massless "on-shell" gluons. The result of the collision will therefore be two heavy baryons flying in opposite directions; later on – possibly in multi-step processes – the baryons may decay into lighter baryons and mesons. Examine the first phase, ie the excitation process. This is how it proceeds: One of the protons emits a gluon, whose transverse momentum is taken to be zero for simplicity, while the longitudinal component is a random variable, its probability density function being given by the structure function of the gluons within the proton. This gluon is absorbed by the approaching other proton. In the next step the protons might change roles.

How do the proton masses change during such an "exchange"? Is it right that low-momentum gluons can produce large resonance masses? What happens in the rest frame of the absorbing proton? How much momentum is transferred to the "gluon-absorbing" proton, when calculated with the usual formulas of inelastic scattering? It is well known that the proton gets fragmented instead of excited, when absorbs a momentum in excess of 1 GeV. Do we step over this limit in the previous example, if we wish to produce a 5 GeV resonance? Can another Lorentz invariant quantity be used for measuring the momentum transfer? Try to elucidate the vaguely defined concepts and statements above. Concentrate on the magnitude of the quantities involved, as well as on matters of principle.

(Gábor I. Veres)

34. Ultra high energy cosmic rays (UHECR) occasionally reach the earth. So far 14 particles (most probably protons) have been detected whose energy exceeded 10^{20}eV. The all-time high particle energy, $\approx 3 \times 10^{20}$ eV was recorded by the detector "Fly's Eye".

A possible scenario for the creation of such particles is this: Along with the cosmic microwave background, the Big Bang theory also predicts the existence of a cosmic neutrino background (approximately 56 neutinos per cc). These neutrinos are essentially at rest. It is suspected that numerous UHE neutrinos reach the earth as well as UHE protons. When these neutrinos hit their stationary colleagues, Z-bosons may be produced – as long as the rest mass of the stationary neutrino is sufficiently large and the cosmic neutrinos are really energetic. The Z-bosons then disintegrate into various particles, among other protons. These protons (and antiprotons) might then be the detected UHECR-particles.

Suppose that the proton produced in the Z-decay takes about 1/100 of the Z particle's energy (in the Z rest frame). Determine the neutrino mass if the all-time champion ($\approx 3 \times 10^{20}$ eV) proton had been created this way, supposing the outgoing proton was moving in the same direction as the incoming neutrino. What is the answer if one takes into account the angular distribution of the protons, $p \propto 1 + \cos^2\theta$, where θ is the angle between the directions of motion of the outgoing proton and the incoming neutrino, measured in the Z rest frame.

The above determination of the neutrino mass is based on the idea that the incoming neutrino energy necessary for Z production depends on the neutrino mass. This rang the bell for Prof. Finga Reen Avery 3,14 – fractional descendent of the great fuzzycist, Finga Reen Avery 0 –, who immediately proposed the following experiment:

Let two neutrino beams collide. The energy of one beam is kept fixed (around the Z mass), and the energy of the other is tuned so that Z bosons are produced in the collision. As demonstrated above, the neutrino mass can be calculated from the two beam energies. Should we recommend that the experiment be realized (supposing that the technical difficulties of producing beams of sufficient luminosity around the Z energy have been overcome)?

(Sándor Katz)

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