THE 31th
– 3nd INTERNATIONAL –
RUDOLF ORTVAY
PROBLEM SOLVING CONTEST IN PHYSICS
2000

The Physics Students' Association of Eötvös University, Budapest, the Roland Eötvös Physical Society and the Hungarian Association of Physics Students proudly announce the traditional, **31th** – and for the third time **international** –

Rudolf Ortvay Problem Solving Contest in Physics,
from November 10, 2000, through November 20, 2000.

Every university student from any country can participate in the Ortvay Contest. PhD students compete in a separate category. The contest is for individuals; solutions sent by groups of students are not accepted. The name, the university, the major, and the university year should be given on the solutions. Pseudonyms and passwords cannot be used: each contestant has to use his/her own name.

The problems can be downloaded from the webpage of the Ortvay Contest

http://ortvay.elte.hu/

in Hungarian and English languages, in html, LaTeX and Postscript formats, from 12 o'clock (Central European Time or 11:00 GMT), Friday, November 10, 2000. The problems will be distributed by local organizers at many foreign universities, too.

Despite all the efforts of the organizers, it may happen that some unclear points or spelling mistakes stay in the text. Therefore it is very useful to visit the webpage of the contest every so often, as the corrections and/or modifications will appear there.

Each contestant can send solutions for 10 problems. For the solution of each problem 100 points can be given.

Each problem should be presented on (a) separate A4, or letter-sized sheet(s). The contestants are kindly asked to use only one side of each sheet. Solutions written by pencil or written on thin copy-paper will not be accepted as these cannot be faxed to the referees.

Any kind of reference may be consulted; textbooks and articles of journals can be cited. Computer programs appended to the solutions should be accompanied by detailed descriptions (what computer language it was written in, how to use the program, which parameters can be set, what notations are used, how to interpret the output figures and graphs, etc.) They can be enclosed on floppy disks, or sent via email to the address given below.

Solutions can be sent by mail, fax, or email (in LaTeX, TeX or Postscript format – or, if it contains no formulae, in normal electronic mail). Please, use basic LaTeX style files. Do not use very special style files unless you include them in your file.

Postal Address: Fizikus Diákkör, Dávid Gyula,
ELTE TTK Atomfizika Tanszék,
H-1117 Budapest, Pázmány Péter sétány 1/A
Fax: Gyula DÁVID, 36-1-3722775
E-mail: dgy@ludens.elte.hu

Deadline for sending the solutions: 12 o'clock CET (11:00 GMT), 20 November, 2000.

Contestants are asked to fill the form available on our webpage after posting their solutions. It will be used for identification of contestants and their solutions. **Without filling the form the organizers cannot accept the solutions!**

The contest will be evaluated separately for each university year, according to the total number of points. The referees reserve the right to withhold, to multiply or to share some prizes. Beyond the money prizes given for the first, second, and third places, honourable mentions and special prizes for the outstanding solutions of individual problems can be awarded. This is why it is worthwhile sending even one or two solutions.

The sponsors of this year's contest are the Roland Eötvös Physical Society, CAT SCIENCE BT, KFKI RMKI, Pro Physica Students' Foundation (Szeged), and physicists István Horváth, Tamás Serényi and Márton Major. Their help is highly acknowledged – and warmly welcome in the future.

The announcement of the results will take place on 7 December. (The place of the event is to be announced later – check it out on the website.) The detailed results will be available on the webpage of the contest thereafter. Certificates and money prizes will be sent by mail.

We plan to publish the assigned problems and their solutions in English language – to which the contribution of the most successful participants is kindly asked. The volume is planned to be distributed all over the world with the help of the International Association of Physics Students, as well as the contestant themselves. We hope this will help in making the contest even more international.

Wishing a successful contest to all our participants,

the Organizing Committee: Gyula Dávid, Attila Piróth, József Cserti

1. A solid wooden ball of density 500 kg/m^3 is tied to a dog's tail. The tail, whose mass and thickness are negligible, is 80 cm long and starts at a height of 40 cm. The dog starts running at a speed of 1 m/s. When it hears that the ball has bumped against the ground, it doubles its pace (NB, Superdog can run as fast as desired).

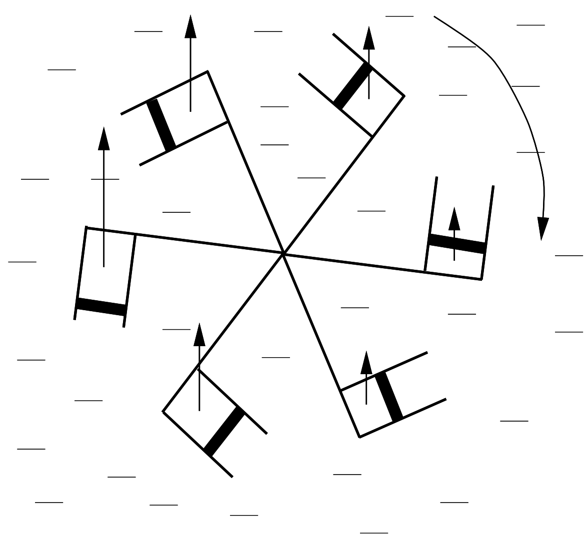
   a) What speed will it reach after a long while?

   b) If the density of the ball may be increased at will, what is the limiting speed that our Superdog will not exceed?

(Gyula Szokoly)

2. Dr. Ivan Knowsalut, employed by the Perpetuum Mobile Telephone Company, keeps bombarding the Patent Office with his perpetual motion machines. Albert Onestone at the Patent Office, unable to cope with Dr. Knowsalut's latest model, asks for the Ortvay competitors' help. The description of the machine is the following:

   "Cylinders, which are fixed to spokes perpendicularly, contain equal amounts of air. The air is closed into the cylinders by easily moving, heavy pistons (see figure). The number of spokes is at least three, the distances



   between the neighbouring spokes are equal. The whole setup is under water, and is able to rotate around a horizontal axis going through its center. The volume of the air, due to the heavy pistons, is increased in the cylinders facing down, therefore the buoyancy exerted on these cylinders (on the left side of the figure) is larger than on the other side. As a consequence, the setup will rotate clockwise."

   Help Albert Onestone to show that the setup will not rotate forever!

(Zénó Farkas – based on Zsolt Bihary's idea)

3. A state-of-the-art version of bungee jumping is called the 'catapult'. The participant is fixed to the ground, the elastic rope is stretched by a crane, and then the fixing is released ('ejecting' the jumper.) Naturally, all this happens on a flat surface. Is there a danger of accident if

   a) the participant is ejected vertically?

   b) the elastic rope is stretched laterally?

(Gyula Szokoly)

4. The interaction between outerspatial litter and artificial satellites is given by the equation

$$\text{litter} + \text{satellite} = \text{more litter}$$

   Estimate how many satellites need to be launched to set up the chain reaction.

(András Bodor)

5. A moving walkway of width d moves at a speed c, while one can walk at a speed v relative to the walkway (which is the same as one's walking speed on the ground.)

   What trajectory should one follow if one wishes to traverse the moving walkway and reach the point across the walkway exactly opposite the starting point? What is the minimum time of traversal if the speed ratio v/c is
   a) greater
   b) less than the golden ratio?

(Gyula Szokoly and Péter Gnädig)

6. One end of a thin, flexible, unstretchable, massless string is clamped, the other is led over a pulley and attached to a massive object (which is hanging on it). A pearl, which can slide without friction, is thread onto the originally horizontal piece of string.

   a) Determine the potential felt by the pearl.

   b) Thread another pearl onto the string. What potential is felt by the two pearls? How do they interact?

(Szabolcs Borsányi)

7. Placing a solid rubber ball (trickball) of radius R on the platform of a (traditional) record player, the record player is turned on. For some time T the platform rotates with a constant angular acceleration β, after which the motor is turned off, and the platform slows down uniformly and stops in time T. Determine the motion of the trickball, if it rolls without slipping throughout the experiment, and does not fall off the platform.

(Péter Gnädig)

8. One end of a massive rope of length ℓ is attached to the vertical axis of a motor rotating at an angular velocity ω, the other end can hang freely. The motor is turned on. After a long period of time has elapsed, the rope rotates as a rigid body with angular velocity ω. Determine (and plot) the shape of the rope for different angular velocities. Discuss the cases $\omega \ll \sqrt{g/l}$, $\omega \approx \sqrt{g/l}$, and $\omega \gg \sqrt{g/l}$!

(Péter Gnädig)

9. An hour-glass is fixed to the end of a weightless rod, and the assembly is swung. The sand keeps trickling. Determine the motion of the system.

(Szabolcs Borsányi and András Bodor)

10. A circular membrane is deformed slightly (approx. 1%), into an elliptic shape. How does the fundamental frequency change? Is it possible to 'hear' this shift?

(József Cserti)

11. Consider atoms of mass m confined in a harmonic oscillator potential $V(\mathbf{r}) = m\omega_0^2(x^2 + y^2 + z^2)/2$. In the hydrodynamical regime, collisions ensure local equilibrium with a given mean velocity $\mathbf{u}(\mathbf{r},t)$, temperature $T(\mathbf{r},t)$, and mass density $\rho(\mathbf{r},t)$. Assuming that the gas can be described as a classical ideal gas, one can search for solutions of the Boltzmann equation in the form $f(\mathbf{r},\mathbf{p},t) = \exp\{-\beta(\mathbf{r},t)\xi(\mathbf{p},\mathbf{r},t)\}$, where $\beta(\mathbf{r},t) = 1/k_B T(\mathbf{r},t)$, and $\xi(\mathbf{p},\mathbf{r},t) = [\mathbf{p} - m\mathbf{u}(\mathbf{r},t)]^2/2m - \mu(\mathbf{r},t)$. In the linear approximation of the Boltzmann equation one obtains the following coupled equations for the mass density, the velocity field, and the pressure:

$$\partial_t \rho(\mathbf{r},t) = -\nabla[\rho(\mathbf{r},t)\mathbf{u}(\mathbf{r},t)] \approx -\nabla[\rho_0(\mathbf{r})\mathbf{u}(\mathbf{r},t)],$$

$$\rho_0(\mathbf{r})\partial_t \mathbf{u}(\mathbf{r},t) = -\nabla P(\mathbf{r},t) + \rho(\mathbf{r},t)\mathbf{f}(\mathbf{r}),$$

$$\partial_t P(\mathbf{r},t) = -\frac{5}{3}\nabla[P_0(\mathbf{r})\mathbf{u}(\mathbf{r},t)] + \frac{2}{3}\rho_0(\mathbf{r})\mathbf{u}(\mathbf{r},t)\mathbf{f}(\mathbf{r}),$$

where $\mathbf{f}(\mathbf{r}) = -\nabla V(\mathbf{r})/m$, $\rho(\mathbf{r},t) = \rho_0(\mathbf{r}) + \delta\rho(\mathbf{r},t)$, $P(\mathbf{r},t) = P_0(\mathbf{r}) + \delta P(\mathbf{r},t)$, and $\rho_0(\mathbf{r})$ and $P_0(\mathbf{r})$ are the equilibrium mass density and pressure, respectively.

   - Given N atoms in the gas at equilibrium temperature T_0, what are the other equilibrium quantities $\mu_{eq}(\mathbf{r}) = \mu - V(\mathbf{r})$, $\rho_0(\mathbf{r})$, and $P_0(\mathbf{r})$?
   - What are the eigenfrequencies of the atomic gas in the presence of small-amplitude perturbations $\delta\rho$, δP, $\mathbf{u}$? (Hint: One can find the eigenmodes in the form $\delta\rho(\mathbf{r},t) = \cos(\omega t + \phi_0)\rho_0(\mathbf{r})\rho_1(\mathbf{r})$, $\delta P(\mathbf{r},t) = \cos(\omega t + \phi_0)P_0(\mathbf{r})P_1(\mathbf{r})$, where $\rho_1(\mathbf{r})$ and $P_1(\mathbf{r})$ are finite polinomials of x, y and z).

(András Csordás)

12. When a towel is folded, a small backward kink at the lower end of the folding line appears very often. What are the criteria for the appearance of such a kink? Use no more than 3 or 4 relevant parameters for the description of the possible cases.

   Try to investigate different methods of folding. Describe the phenomenon theoretically and numerically.

(Illés Farkas)

13. Granulated sugar is put and stirred up in a cylindrical cup of tea, then the spoon is removed. Where does the sugar accumulate? The answer is completely different if the cup rotates together with the water and the sugar is just poured into it from above – in which case the water might need to be disturbed by a spoon to ensure the free motion of the sugar grains. Examine the phenomena experimentally and explain the observations theoretically.

(Dániel Barna)

14. A cubic tank with heat insulating walls is filled up with a gas of temperature T_1, and is connected to an identical tank that is filled up with a gas of temperature T_2. A small circular hole of diameter d (much smaller than the mean free paths) is punched into the wall separating the two tanks.

a) What will the pressure be in the two tanks after a long while, if they are kept (through some external agent) at temperatures T_1 and T_2, respectively?

b) What resultant force is exerted on the walls of the tanks by the gas? What is the origin of this force?

c) How much work can the system do upon taking up heat Q from its environment?

(Péter Gnädig)

15. Determine the efficiency of a heat engine with two heat reservoirs (of temperature T_0 and T_1), if heat transfer between the engine and the reservoirs is due to heat conduction, and maximum performance is assumed. The law for heat conduction is linear:

$$\frac{dQ}{dt} = R(T_1 - T_0).$$

(Katalin Martinás)

16. Consider a conic cogwheel that can rotate without friction in empty space and whose axis is perpendicular to the observer's line of sight. The thermal radiation emitted by the part of the cogwheel moving at the largest circumferential velocity towards the observer is blue-shifted (Doppler effect), thus its momentum gets larger. Reflecting this radiation by a mirror back to the smaller-diameter part of the cogwheel receding from the observer, this part of the cogwheel takes up more momentum by absorbing the radiation than it loses by emission. Therefore the cogwheel receives a net torque, which leads to a runaway increase of the angular velocity. Or ...?

(Zoltán Bódi)

17. A square lattice of identical resistances (1 ohm along each edge) is wrapped around an infinite cylinder in such a way that N resistances are placed along the cross sectional circle of the cylinder. What are the resultant resistances between two adjacent lattice points in the "axial" and in the "circumferential" directions?

First consider the cases $N = 2$, $N = 3$, and the limit $N \to \infty$.

(Péter Gnädig)

18. The edges of a cube are made of a thin wire of uniform specific resistance. A current of 1 A flows into one of the vertices, and out at another vertex at the other end of a face diagonal. (The connecting wires are long straight conductors, the lines of which cross the centre of the cube.)

What is the magnetic field strength at the centre of the cube?

Generalise the problem for the case of other regular polyhedra, e.g. determine the magnetic field strength at the centre of a regular dodecahedron for different relative positions of the entering and exiting currents.

(Gyula Radnai and Péter Gnädig)

19. One of two capacitors of capacity C is charged to voltage U, the other is uncharged. The capacitors are connected by two parallel wires of length L and cross sectional area S, separated by a distance d. The resistance of the wires and the capacitors is negligible, and L is much larger than the size of the capacitors. Determine how the energy of the system changes with time.

(Péter Hantz)

20. Two dielectric sheets (of large surface area and negligible thickness) of permittivity ϵ are placed parallelly at a distance d from each other in empty space (d is much less than the horizontal dimensions of the sheets). Determine the frictional force developed between the two sheets if one is set in motion with a relative speed v.

(Titusz Fehér)

21. In a toroidal shaped wave guide of length $2R\pi$ the electric field satisfies the wave equation $\ddot{E}_z = E_z'' - \kappa^2 E_z$ in the direction of propagation for some modes; the derivative E_z'' is with respect to the direction of the wave guide. A Gaussian wave packet of width σ enters the system at $z = 0$. Suppose that $\sigma \ll R$ and $\kappa R \gg 1$ $(c = 1)$. The electric field $y(t) \equiv E_z(t, z = 0)$ is continuously measured at $z = 0$.

a) Write up the function $y(t)$, and show that the transient period is followed by a steady state (though rather complicated) oscillation.

b) From the time evolution of the early transient period determine the functional form of the decay of $y(t)$ with time.

c) Demonstrate that the stationary phase solution can be approximated by the simple harmonic solution of the equation $\ddot{y}(t) + \eta\dot{y}(t) + \omega^2 y(t) = 0$. Determine the best fitting values of η and ω by taking the long-time average of the products of $y(t)$ and its time derivatives.

(András Patkós and Szabolcs Borsányi)

22. Place a "very" massive charge at the origin $\mathbf{r} = \mathbf{0}$. A plane electromagnetic wave is incident from $x = -\infty$ with an electric field given by $E_y(\mathbf{r}, t) = f(x - ct)$, where $f(x)$ is non-zero only in a finite domain. Describe how the wave interacts with the charge, and how the energy and momentum of the charge and the field develop in time. What approximations did you use (e.g. what does "very" massive mean above)?

(Titusz Fehér)

23. Consider a superconducting ring, in which a quasi-stationary current is set up. Measuring the intensity of the current in a year's time a decrease of less than 0.1% is found. Give an (upper) estimate on the specific resistance of the superconducting ring. (Suppose the ring is of toroidal shape with a radius of 1 cm; the radius of the cross section of the superconducting wire is 1 mm.)

(Titusz Fehér)

24. Sitting on the side of an aircraft farther from the sun, and watching its shadow cast on the clouds, one may see rainbow-coloured rings around it. Determine the deviation of light due to the pressure differences caused by the aircraft for different colours.

If clouds are absent, it is practically impossible to make out the coloured rings on the ground, since it is much farther than the clouds, therefore the intensity of the rings is way too small. However, one can notice that the intensity of light around the much smaller shadow changes significantly with its radius, i.e. the altitude of the aircraft. Why?

(János Török)

25. Ultra high energy cosmic rays (UHECR) occasionally reach the earth. So far 14 particles (most probably protons) have been detected whose energy exceeded 10^{20}eV. The source of these particles is unknown. An interesting feature is, however, that two of these particles have been detected from the same direction, up to the resolution of the measurement. (If a threshold energy of 4×10^{19}eV is considered, higher multiplets are found as well: 7 doublets and 2 triplets out of 92 events.)

a) Determine the probability that such multiplets are detected (for both 14 and 92 overall events), if the arrival of the particles at the Earth is equally probable from all directions, and the angular resolution of the measuring apparatus is 3 degrees.

b) An alternative explanation of the origin of the multiplets can be the existence of a finite number of pointlike sources of UHECRs; particles detected in the same direction are assumed to have originated from the same source. Determine the most probable value of such sources within a sphere of radius 50 Mpc, assuming the following:

- Sources are distributed evenly; on the average there are N sources within a sphere of radius 50 Mpc.
- All such sources emit protons with the same intensity; the emissions follow a Poisson distribution.
- Protons tend to lose energy; having traversed distance r, the number of protons of energy higher than 10^{20} eV is proportional to $e^{-r/R}$, where $R \approx 50$ Mpc.
- Above 10^{20}eV 14 particles have been detected to date, of which 2 arrived from the same direction.
- The detector is located on the Northern Hemisphere, at a latitude of 45 deg. Take into account that the detector cannot observe the whole sky at the same moment.

(Sándor Katz)

26. Consider a physical process in which one characteristic parameter is measured (e.g. the strength or the duration of an earthquake). One wishes to test the feasibility of a theoretical model for the distribution of the quantity in question by performing 1000 measurements (e.g. the gaussianity of the distribution).

What methods would you use to decide whether the 1000 measured data $x_1, x_2, \ldots x_{1000}$ are compatible with the theoretical probability distribution function $f(x)$?

According to an other theory, two independent processes can trigger an earthquake (to stick to this very example). In this case the characteristic parameter (as above) may be described by two (possibly) different PDFs.

More specifically, consider two Gaussian distributions, whose mean values, as well as their standard deviations are different. What method can decide whether the real PDF behind the 1000 measured data can be reliably characterised by a Gaussian distribution, or the assumption of two independent Gaussians should be used?

(István Horváth)

27. One is standing at the origin of the coordinate system with a searchlight (capable of emitting a strong parallel beam of light) in her hand. At a distance d measured in the direction of the y-axis a cube passes in the positive x direction with a uniform speed V. The faces of the cube are parallel to the coordinate planes in its rest frame. As it is well known (see, e.g. Taylor and Wheeler, *Space-Time Physics*) if a body passes by an observer at a relativistic speed, it will not appear to be contracted in the direction of the motion (as would follow from the naive interpretation of Lorentz contraction) but rather rotated; more specifically the rear face will be rotated towards the observer. Now the observer shines a parallel beam of light on the cube by directing her searchlight in the direction of the y-axis. What will be the intensity of illumination on the side face parallel to the direction of motion closest to the observer, and on the rear face turned towards the observer? Isn't there some contradiction or paradox here? In which sense has the cube turned in reality? Describe everything in the rest frame of the cube as well.

(Gyula Dávid)

28. Construct the general theory of NON-relativity (GNR), i.e. take the basic idea of general relativity, according to which

a) gravitational phenomena are caused by the curvature of space (or spacetime) due to the presence of material, and

b) the trajectory of a body (acted upon only gravitationally) is the straightest possible line in this manifold.

On the other hand, discard phenomena investigated by special relativity (e.g. the unsurpassable limit and the invariance of the speed of light), i.e. accept the Galilean-Newtonian model for gravitationless spacetime.

What is the form of the equations of motion and the gravitational field equations in GNR? What known phenomena can be accounted for in this framework (at least at a qualitative level)? What phenomena cannot? Propose an experiment to decide whether GR or GNR is the correct theory. Are there any phenomena that GNR can handle better (or at least in a more reassuring fashion) than GR? Are there black holes, white holes, or a Big Bang in GNR?

(Gyula Dávid)

29. Quantise the system given by the Hamiltonian

$$H(p_x, x, p_y, y) = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{m}{2a}(x-y)^2 + \frac{1}{2}m\omega^2(x^2+y^2) + \frac{\lambda}{24}(x^4+y^4).$$

Determine the vacuum state and the first excited state. Calculate the energy of one and two-particle states to order λ^2. Write up the effective equation of motion for the expectation values of x and y.

(Szabolcs Borsányi)

30. Consider an electron in a rectangular box divided in two parts parallel to one of its sides. The electron's effective mass is different in the two regions. The wave function is zero on all boundaries. What is the matching condition for the wave function at the interface of the two regions? Determine the energy eigenvalues.

(József Cserti and Géza Tichy)

31. In the EPR experiment, pairs of spin-1/2 particles are emitted by the source, in singlet state W_s. We perform spin measurements along directions $\mathbf{a}$, $\mathbf{a}'$, and $\mathbf{b}$, $\mathbf{b}'$. The assistants can freely chose between the two possible measurements (by tossing a coin, for example). According to QM, the probability of the outcome "up" in an $\mathbf{a}$-measurement is $p(A) = tr\,(W_s P_A) = 1/2$. Similarly, the probability that both the outcome of an $\mathbf{a}$-measurement and the outcome of a $\mathbf{b}$-measurement are "up" is:

$$p(A\&B) = tr\,(W_s P_A P_B) = \frac{1}{2}\sin^2\left(\frac{1}{2}\measuredangle\,(\mathbf{a},\mathbf{b})\right).$$

In a typical scenario the angles are coplanar, $\measuredangle(\mathbf{a},\mathbf{b}) = \measuredangle(\mathbf{a},\mathbf{b}') = \measuredangle(\mathbf{a}',\mathbf{b}') = 120°$, and $\measuredangle(\mathbf{a}',\mathbf{b}) = 0$. Consequently, the probabilities are $p(A) = p(A') = p(B) = p(B') = 1/2$, and $p(A\&B) = p(A\&B') = p(A'\&B') = 3/8$, while $p(A'\&B) = 0$. This experiment has been performed many times in reality, and the results were in good agreement with the QM calculations.

Construct a possible laboratory record for such an experiment! That is, fill out the following table in such a way that the relative frequencies calculated from the table reproduce the above probabilities:

n	A	A'	B	B'	$A\&B$	$A\&B'$	$A'\&B$	$A'\&B'$
1	1	0	0	1	0	1	0	0
2	1	0	1	0	1	0	0	0
3	0	0	1	0	0	0	0	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

(László Szabó)

32. A pair of particles emitted by a source in opposite directions is registered by two detectors, A and B, far from the source and each other. Each detector has a switch with two settings labelled 1 and 2, and two flashes, a red (R) and a green (G) one. After the particles have left the source, but before they have arrived at the detectors, one randomly and independently sets the switch on each detector to either of its two positions. Triggered by the particle, one of the lamps on each detector flashes. Repeating the experiment several times, the following observations are made:

a) If the detector switches are set differently, the detectors never flash both green, i.e. A1G, B2G and A2G, B1G never occur.

b) If the detector switches are both set to 1, the detectors never flash both red, i.e. A1R, B1R never occurs.

c) If the detector switches are both set to 2, the detectors occasionally flash both green, i.e. A2G, B2G sometimes occurs.

Show that the explanation of these results leads to contradiction if one makes the following reasonable assumptions:

(i) the detectors do not influence each other;

(ii) the detector may respond only to some feature of the particle triggering it, and by no means to some property of the other particle (which flew to the other, faraway detector);

(iii) the correlations between the properties of the particles are not affected by the setting of the switches.

What circumstances make these assumptions reasonable? Supposing that the pairs consist of two-state particles, give a quantum mechanical state of the pairs which would produce the observed results. Assume that each emission results in the same quantum state of the pair. What is the maximum possible probability of the events A2G, B2G given in point c?

(Mihály Benedict)

33. Every well educated child knows that the bound state of two fermions is a boson. Red Neck has notoriously been playing truant at kindergarten, and so he has some doubts. He thinks: "There are two fermions in a hydrogen atom, so it should be a boson. On the other hand, I can easily put 100 bosons in the same state, while I would certainly fail with 100 H atoms, since each H atom contains an electron inside, and these cannot be in the same state!" To teach Red Neck a new tune, consider the following non-relativistic quantum-mechanical model. Let

$$\mathcal{H}_{2\mathrm{B}} = \{\psi \in \mathcal{L}^2(\mathbb{R}^3 \times \mathbb{R}^3, \mathbb{C}) | \, \psi \text{ symmetric}\}$$

be the Hilbert space of two identical spin-zero bosons, in which rotations, spatial translations, and Galilean transformations act as usual:

$$\psi \to \psi'(x_1, x_2) = e^{imv(x_1+x_2)}\, \psi\left(F^{-1}x_1 - a, F^{-1}x_2 - a\right).$$

Here F is a rotation, a is a spatial vector (for spatial translation), v is a velocity vector (for the Galilean transformation), and m is the mass of the bosons. Let us denote this representation by $U_{2\mathrm{B}}$.

In a similar manner, one can define the Hilbert space $\mathcal{H}_{4\mathrm{F}}$ of four identical spin-half fermions and a unitary ray representation $U_{4\mathrm{F}}$ of the above transformations.

Without giving an explicit form for the Hamiltonian, it is assumed that it contains two-fermion interactions only, and that there exists a bound state of two fermions in which the total spin and the relative orbital angular momentum is zero. Furthermore, it is assumed that in the relative coordinates such a ground state is unique.

The mathematical formulation of the statement that there are states in which the four-fermion system looks as if it was made up of two spin-zero bosons is that there should be a closed linear subspace M of $\mathcal{H}_{4\mathrm{F}}$, for which $U_{4\mathrm{F}}$ can be restricted, and is equivalent to $U_{2\mathrm{B}}$. Give such an M and the unitary (or antiunitary) operator V making the equivalence. What is the form of the two-boson state $\psi(x_1, x_2) = \varphi(x_1)\varphi(x_2)$ in $\mathcal{H}_{4\mathrm{F}}$?

(Mihály Weiner)

34. The Hamiltonian (operator/function) of a monatomic linear chain is given by

$$\mathcal{H} = \frac{1}{2m}\sum_{n=1}^{N} p_n^2 + \frac{\mu^2}{2}\sum_{n=1}^{N} u_n^2 + \frac{D}{2}\sum_{n=1}^{N-1}(u_{n+1}-u_n)^2 \ ,$$

where p_n and $u_n = x_n - na$ are the momentum and the displacement of the nth atom, respectively (a is the lattice constant). An external force $f = f_0 \cos\omega t$ is applied to the first atom of the chain.

Find the amount of energy transferred to the chain in unit time.

(László Sasvári)

35. Consider a plasma cloud consisting of ions of charge Z, atomic mass M and electrons in a homogeneous magnetic field B. The magnetic field is directed along the x direction. The charged particles move on Larmor trajectories around the magnetic field lines. If the Larmor radius is small compared to the plasma cloud size, particle motions can be considered as one-dimensional, in the x direction only. At time $t = 0$ the cloud is confined to a rectangular region of length L in the x direction and width d in the other two directions. Inside the rectangular region the ions have uniform density n_i, the electrons $n_e = Zn_i$, thus the plasma is neutral. For simplicity, assume that the ions have zero temperature ($T_i = 0$), while the electrons have some initial temperature T_e. Assume that the Debye length $\lambda_D = \sqrt{\epsilon_0 k T_e / n_e q_e^2}$ is much smaller than the characteristic spatial lengths. If the density of the plasma is in a range where the collisional mean free path of the particles is much longer than the size of the cloud, the collisions between the particles can be neglected. Such conditions are quite common in high temperature laboratory plasmas.

What happens to the plasma if it is left alone at $t = 0$? What is the short and the long time-scale behaviour? Try to develop a qualitative picture of the processes first.

(Sándor Zoletnik)

36. "Billy, why didn't you clean your room?"

"Because I read that all activities increase the disorder of the world, i.e. the entropy of the Universe. And I do mind what world I would leave behind."

Is Billy right? Should one clean up if one thinks of the future seriously?

(Katalin Martinás)

37. Robinson Crusoe reaches a seemingly desert island, but suddenly he is met by a tribe of cannibals, and greeted by the chief. "*Voilà*, here is a one-half spin. You can measure on it whatever you wish. Then it is our turn, and we shall measure it randomly in the x, y, or z direction. Then we all go to sleep, and tomorrow morning you can perform one more measurement. If you can tell us whether the result of our measurement was $+$ or $-$, you are free. Otherwise, you will end up in the big cauldron!"

a) By what strategy can RC maximise his chances for survival?

There, there, no problem, RC thinks to himself, but the chief goes on. "I have three naughty kids from my ex-squaw, Decoherence: Sigmaex, Sigmawhy and Sigmazed. No matter how many guards keep their eyes on the Holy Tepee, one of these good-for-nothings will try to sneak in. And if she gets in, she will operate on the spin according to her name. But I quite like you, therefore, if you wish, I can put an extra guard by the entrance, and then they only have a 60 per cent chance to get in."

b) What should RC answer to the chief? What is the best strategy now, and what are the chances for survival?

(Zsolt Bihary and Attila Peták)

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